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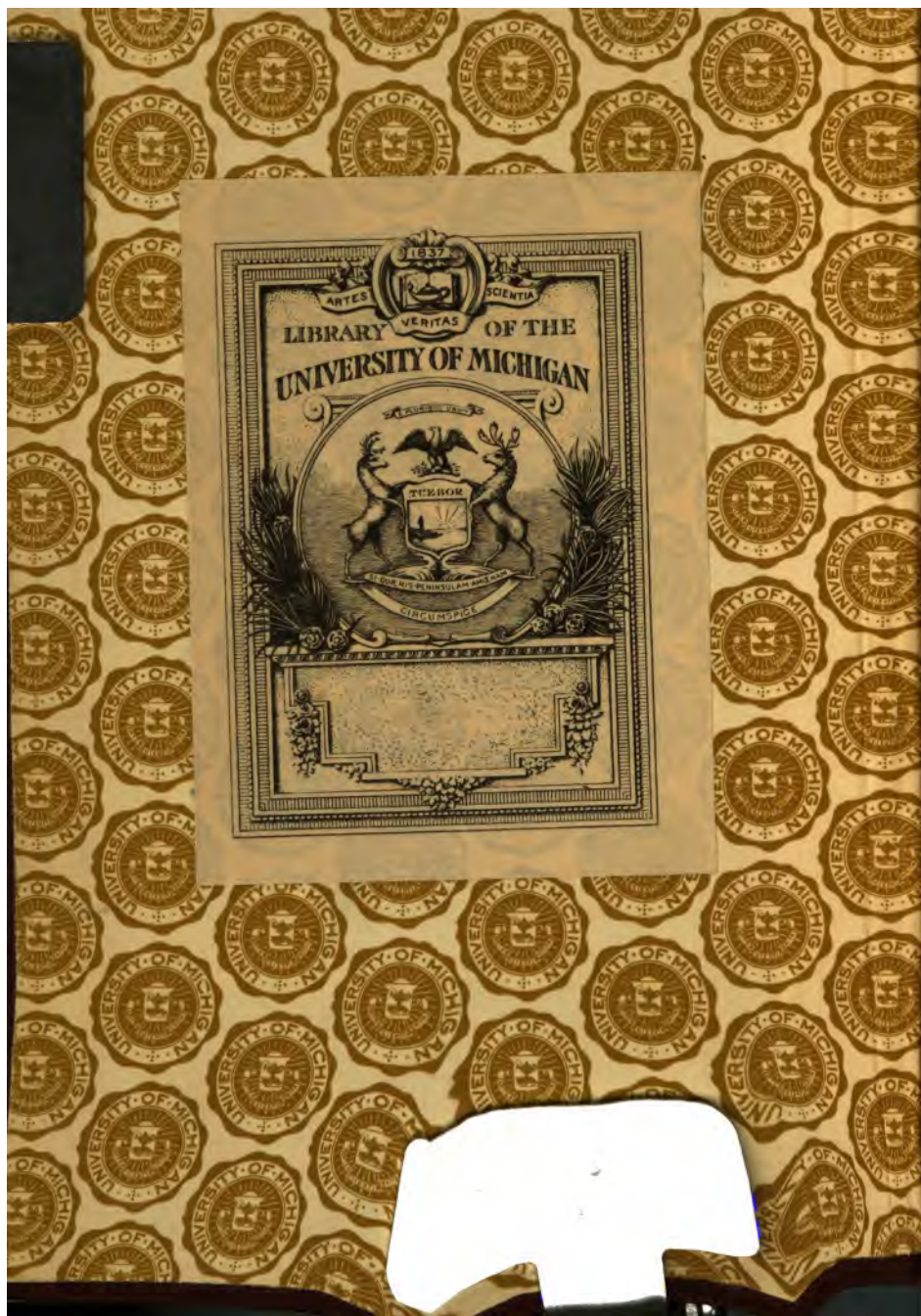
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HOW TO TEACH PRIMARY NUMBER

A COURSE OF
STUDY AND A MANUAL
FOR TEACHERS

BY

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PREFACE

The number work of the first four years lays the foundation for future work, and a pupil's progress in the intermediate and advanced grades will depend very largely upon the foundation that he gets in these early grades.

On account of the nature of the work, and his inability to get from a textbook a complete meaning of the facts and processes, the pupil's understanding of and attitude toward the subject of number depends upon the teacher to a much larger degree in the earlier than it does in the more advanced grades. Unless the teacher of these lower grades understands how to *develop* the fundamental facts and processes; how to grade her work, and *drill* in a way to give the pupil an automatic control of them; and how to teach him to *apply* them to situations that arise in his life, the pupil will be greatly hampered throughout the whole course.

Since so much of the work of these early grades is independent of a textbook, this book is prepared as a course of study and a manual for the teacher of the first four grades. It aims to give carefully planned directions as to how to treat the three phases of the subject—presentation, drill, and applications. It not only shows how to present the fundamental facts and processes, but it provides well-graded drills that will, if followed, insure proper attention to *all* facts of each series. Throughout

the book the games and other devices best suited to the special facts under consideration are given. In addition to this, many typical problems are given and many sources of problem material are suggested.

A manuscript from which this book was finally made was prepared about three years ago to supplement the author's book *The Teaching of Arithmetic* for his students who were preparing for work in the primary grades. Copies of the manuscript were used for reference by the author's students in the summer sessions of The Cleveland School of Education, and for a year or more the book was used in manuscript form by primary supervisors in several schools. Many of those using the manuscript have urged the author to publish it in book form. It is offered, then, in the belief that the methods and devices will help the teacher in planning her work, and thus help the pupil to get a proper training as well as pleasure in his work.

The author takes pleasure in acknowledging his indebtedness for many helpful suggestions from the following: Miss F. Avis Coultas, Primary Supervisor in the Cleveland (Ohio) Public Schools, who used the manuscript for nearly two years; Miss Nellie I. Jacobs, Demonstration Teacher in the second grade of the Montclair (N. J.) State Normal School, who used it in her work in the second grade; and to Miss Orpha E. Worden, Teacher of Method in Arithmetic in the State Teachers' College, Detroit, Mich., who carefully read the manuscript when it was first prepared.

May, 1922.

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HOW TO TEACH PRIMARY NUMBER

CHAPTER I

GENERAL PRINCIPLES AND SUGGESTIONS

THREE PHASES OF TEACHING

In teaching the primary number facts and processes, there are three distinct phases of the work that are of equal importance; but not, of course, requiring an equal amount of time. These are: (1) the first presentation of the fact; (2) drill upon it until the pupil has an automatic control of it; and (3) its use in simple everyday applications to give the pupil a full appreciation of its meaning and use.

In the matter of time, the second of these phases requires the most. However, the other two must not be slighted in the least. (It is not uncommon to find children who know the facts at sight or by sound and yet who have no notion of the meaning or use of them.) I once asked a child in the fourth grade, "How many are five 3's?" She looked at me in bewilderment, then asked, "Do you mean *and* or *times*, or *less*?" And then she said, "5 and 3 are 8; 5 times 3 are 15; 5 less 3 are 2."

Unless the child knows the meaning of a fact and of the language used, the fact is of no value to him, for he cannot use it. This is brought out through the development of the fact, and by using it in concrete life situations.

THE IMPORTANCE OF PRESENTATION

The importance of the presentation is twofold:

First, through the presentation, the child knows how to find the fact for himself if he forgets it before getting an automatic control of it.

Second, through the presentation, he sees the meaning of it and will thus know how and when to use it. It must not be thought, however, that the mere presentation is sufficient to show clearly its full meaning and use. This is brought out more completely through the application to simple problems that arise in the child's life. The methods of presenting the various facts and how to use them in problems are shown later.

✓ THE IMPORTANCE OF DRILL

The importance of drill can hardly be overestimated, and yet mere drill without sufficient attention to the other two phases is useless. Through drill the child gets an automatic control of the facts and does not have to resort to counting to find a needed fact. But this alone does not insure the child's understanding of them, or his ability to use them in real situations.

The drill must be both *sight drill* and *oral drill*. That is, the child must be able to recall a fact from seeing the figures or from hearing them spoken, or from merely "thinking" them as when he sees them written out in words or has occasion to "think" them in order to answer some personal question that arises.

THE IMPORTANCE OF THE PROBLEM

The importance of the problem in the early number work is to give a clearer meaning to the fact. At the same time, of course, the problem gives drill needed to make the facts automatic; leads to an appreciation of the use and need of the facts; and thus furnishes a motive for learning them. The problems, however, should be of a type that leads to the habit of using arithmetic. Hence they should be of a type that answers a real need, and not merely requires a process. Thus "John had 8 marbles and lost 3 of them. How many has he left?" is not likely to answer a need. But "John had 8 marbles and lost all but 5. How many did he lose?" may answer a real need.

THREE EDUCATIONAL PRINCIPLES INVOLVED

All teaching should be based upon the following principles that should become a part of every teacher's creed.

- ✓ I. *Knowledge to be real must be founded upon the actual experiences of the individual learner.*

✓ II. *Knowledge to be retained must be given an opportunity for use.*

✓ III. *A necessary condition for learning is that the process be self-actuated through motive or interest.*

Hence, the things taught must be kept within the child's actual experiences and made to minister to his needs. When we try to teach him by selecting material from the adult world with which he has had no experiences, we are attempting the impossible.

USING ARITHMETIC IN ALL SCHOOL ACTIVITIES

There would be no object in teaching reading, writing, or spelling unless the pupil gets the habit of using his knowledge outside of the classroom. And so the knowledge of number is of no value unless the pupil gets the habit of using it. In general, the problems of the text are made for the pupil. He gets but little practice in formulating problems for himself or the habit of raising questions about number relations met outside of the class period. The chief aim of the teacher, aside from fixing the facts and processes, should be to develop the habit of using arithmetic wherever quantitative relationships arise that need it. The reading lesson may refer to numbers in which a fuller meaning of the story may require a comparison of them. If so, the pupils should be taught to stop and make such a comparison. A date may occur and the pupils should be

taught to ask, "How long ago was that?" History, geography, nature study, "how we are clothed, fed, and sheltered," and many other school activities furnish a need of number. In all such work, the pupil should be taught to formulate his own problem and hunt the data required to answer it. Some of the problems will have to be formulated by the teacher, and she will have to raise questions that suggest others. But through such work the pupil gradually comes to ask questions for himself.

Stories involving number may be made up to develop the habit of raising questions about number relations. The stories may be retold by the pupil in a way to make the quantitative side clearer by showing the needed relations, thus:

"Mary and Nell gave a doll's party. Mary brought six dolls and Nell four. This was enough for a nice large party. When dinner was served for the dolls, Mary could find but seven plates. So Nell ran home and brought enough so that each doll could have a plate. Mary had four doll chairs and Nell had three. So some of the dolls had to sit on a box while they ate dinner."

Such stories may be extended to any length required. When retold by the children they will fill in the exact number of plates brought by Nell, the number of dolls that sat on the box, etc.

Children may thus be led gradually to see how ~~number~~ number must be used to meet their daily needs.

✓ NUMBER WORK A MOST ENJOYABLE EXERCISE

When properly taught, there is no more enjoyable exercise for children than their number work. It can be made as fascinating as their play. There are so many interesting games in which it may be used, so many little problems that the child must solve in his own little world, that when properly planned there isn't a dull moment in the whole course in number work. It is the purpose of this little monograph to show how this can be done.

CHAPTER II

THE WORK OF THE FIRST YEAR

Following the principle that the work of the school should be based as nearly as possible upon the child's interests and needs, there is a rapidly growing tendency to defer formal number work until the second school year. However, there are needs of counting, of expression, and of measuring that naturally arise. (In the work, then, of the first school year, it is recommended that there be no definite time allotment for arithmetic. It should be taught incidentally as need of it arises.) While the needs differ somewhat according to locality and environment, there are general needs that will be outlined and discussed in this chapter. The teacher, however, should not be confined to this outline if other real needs arise.

SUBJECT MATTER

1. Reading and writing numbers to 200 (based upon finding a page in a book), or farther if the child needs the knowledge to find a house number or a telephone number.
2. Counting by 1's, 5's, and 10's to 100.
3. Reading Roman numerals to XII on a clock face.

4. The meaning of dozen.
5. Common measures, as: inch, foot, yard; pint, quart, gallon; pounds and ounces.
6. Coins: penny, nickel, dime, quarter, and half dollar.

ACTIVITIES REQUIRING NUMBER

1. The number of children in a class or in a game.
2. The number of chairs or desks needed for the class.
3. The distribution of books or other class material.
4. Finding a page in their books.
5. Measuring, to find how tall.
6. Telling the time.
7. The cost of toys, rides, lunches, etc.
8. Interpreting the meaning of common measures spoken of at home, or used in school activities.

ROTE OR RHYTHMIC COUNTING

The first unit of instruction should be to count to ten. It is essential, of course, that the child can name the numbers in their proper order or sequence. This can be accomplished more quickly by drilling upon saying the names of the numbers in proper order without actually counting things. That is, the child rhythmically gives the names of the numbers in their proper order without actually knowing what the names mean. As a help in getting the proper sequence, little rhymes are useful, such as:

One, two, three, four, five,
 I caught a hare alive;
 Six, seven, eight, nine, ten,
 I let him go again.

And,

One, two,
 Buckle my shoe.
 Three, four,
 Shut the door.

Five, six,
 Pick up sticks.
 Seven, eight,
 Lay them straight.

Nine, ten,
 See a fat hen.

Or,

One little, two little, three little Indians,
 Four little, five little, six little Indians,
 Seven little, eight little, nine little Indians,
 Ten little Indian boys.

RATIONAL COUNTING

(When the child can call the names of the numbers in proper order, he should be taught the meaning of these names through counting objects—rational counting. That is, through counting things about the room, he sees the meaning of the names he has learned.)

In having a pupil count objects, the teacher should exercise care that he does not get the idea that the third one of a group is three; the fourth one, four; and so on. This is avoided by presenting groups and raising the question of "How many?" and finding it through counting. Thus presenting three

things, she will ask, "How many?" The pupils, touching or pointing to each in order, will say, "One, two, three; there are three of them." Make use of objects in the room. Ask how many windows, how many books, how many pencils, how many chairs, etc., until the children can apply their rhythmic counting to finding "how many." Use objects in which the children are interested, and about which they really have a desire to know "how many" instead of using splints, beads, etc., made up to sell for such purposes. The child thus gets (1) the *serial* meaning of a number; that is, that five is one more than four or one less than six; and (2) the *group* meaning; that is, that it is the name of the number in a group, as five is the number in |||||.

THE NAMES OF THE FIGURES

Symbols should not be used until the corresponding number idea is clear to the child. But before any of the primary number combinations are taught, the pupils should recognize the names and meanings of the figures. Thus, they should associate the figure with the name and with the objects.

One way to teach the meaning of the figures, through a little matching game, is as follows:

Take thirty cards—ten containing the figures from 1 to 10, one on each card; ten containing the names, "one," "two," "three," and so on; and ten

others containing objects, as dots or pictures. These cards are distributed among the pupils and, as a pupil is named, he runs to the front of the room and shows his card. Other pupils match the card by coming up and standing in line. Thus, if the pupil named has "five," those having "5" and :: "match" him.

Pupils may also play the game alone by taking all thirty cards and building up ten piles by matching the cards. Thus, they deal out the cards, laying them upon the table or desk, placing those that match upon each other.

LEARNING THE FIGURES THROUGH GAMES

Besides the game given above, pupils may be taught the names of the figures very quickly, through calling the scores made in Ring Toss, Bean Bag, and other scoring games, and through charts giving the prices of articles. Such charts as "A Toy Shop," "A Fruit Store," "The Bakery," etc., may be made, having the prices marked from 1 to 10 inclusive. The pupils soon learn the names of the figures by giving the price of any article designated.

COUNTING FROM 10 TO 20

Since our number system is based upon a scale of ten, the first unit of instruction is to count from one to ten. With the exception of eleven and twelve, the rhythm of the following groups depends

upon the rhythm already known; hence the use of this fact will make the learning of the remaining units of instruction comparatively easy. That is, the sounds of "three," "thirteen"; "four," "fourteen"; etc., are so similar that the pupil easily sees that by adding "teen" to the order of counting that he already knows, he can count from

thirteen to nineteen inclusive. Eleven and 3 13
twelve must be taught separately as there 4 14
are no sound-aids to help the memory. 5 15

It is a help in learning to count to 20 to 6 16
teach the written form along with the count- 7 17
ing; for the eye aids the ear in getting the 8 18
new counting from the old. Thus, the 9 19
rhythmic and rational counting, and the
written symbols may all be taught at the same time,
one aiding the learning of the other.

COUNTING BY 10's TO 100

This is the next unit of instruction and is easily accomplished by making use of the similarity of sound between "two," "twenty"; 2 20
"three," "thirty"; "four," "forty"; "five," 3 30
"fifty"; and so on. Thus the child sees that 4 40
by adding *ty* to the numbers that he knows, 5 50
he has the new names. The written forms 6 60
aid in fixing the names. 7 70

To rationalize counting by 10's, let the 8 80
pupils count either real or toy dimes; saying, 9 90

10, 20, 30, and so on, as they count them. Also give such problems as, "What will — pencils cost at 10¢ each?" having the pupils find by counting 10, 20, etc. In this way, pupils soon learn to count by 10's, which is essential before they can count by 1's beyond 29.

COUNTING BY 1's TO 100

The next step in counting is to learn to fill each decade with one, two, three, and so on; as twenty-one, twenty-two, twenty-three, etc. Let the pupils count the number of pupils in the class or in a game; the number of desks or chairs; the books or pencils; or anything in which they are interested in finding the "how many."

A Counting Chart

The following chart will help to fix the order of the number scale. As will be observed, it may be used in counting by 1's or by 10's.

0	10	20	30	40	50	60	70	80	90
1	11	21	31	41	51	61	71	81	91
2	12	22	32	42	52	62	72	82	92
3	13	23	33	43	53	63	73	83	93
4	14	24	34	44	54	64	74	84	94
5	15	25	35	45	55	65	75	85	95
6	16	26	36	46	56	66	76	86	96
7	17	27	37	47	57	67	77	87	97
8	18	28	38	48	58	68	78	88	98
9	19	29	39	49	59	69	79	89	99

COUNTING BY 5's TO 100

The counting already given is essential. Each step was to enable the child to count by 1's to 100. Counting by 5's is not essential but very convenient. Thus, in finding how many minutes past or till a certain hour, the child counts 5, 10, 15, etc., as he looks at the five-minute spaces on a clock face. Or, more often he may need to count his money by nickels. Let the class first count rhythmically, simply saying 5, 10, 15, 20, 25, and so on. Then using either real or toy nickels, count the number of cents in any number of nickels to 20 of them. Drill may be given by raising such questions as, "At 5¢ each, how much will — oranges cost?" To answer, make as many marks on the blackboard as there are oranges, and then count them, saying 5, 10, 15, etc. Or, with the class arranged five abreast, count them by 5's. Many other real occasions for counting by 5's will present themselves, and pupils will easily learn to count in this way.

COUNTING BY 2's

It is very questionable whether it is worth while to count by 2's in this grade. It depends upon whether a need of counting things grouped in pairs arises often enough in the real activities of school, play, or home. If such a need justifies the teaching, it is easily done, first counting rhythmically, then rationally, taking two each time a count is made.

TEACHING THE COMMON MEASURES

There is no value in teaching the abstract "tables" of measure; that is, in requiring pupils to remember the number of units of one order that make one unit of the next higher order. The value comes from being able to think in terms of these measures. Hence the actual measures should be seen and used by the children in real situations. To get an idea of "how much" a pint, quart, or gallon is, common measures about the house are more valuable than the odd-shaped so-called "standard" measures of the schoolroom. That is, a quart or pint milk bottle, a gallon bucket, a gallon oil can, etc., should be used in connection with the measures of capacity, usually found in the schoolroom.

In getting an idea of a pound or an ounce, bags of common commodities weighing a pound or an ounce should be handled by the children. Also bags of 5 and 10 pounds of common commodities give not only a notion of "how heavy," but show the eye the bulk of these that it takes to make a certain weight.

In using the measures of length, the pupils should measure all sorts of heights and distances, and learn to estimate length with a fair degree of accuracy. They like to measure and compare their own heights.

READING AND WRITING NUMBERS

When a child first learns to read or write a number, the place-value feature of our system of nota-

tion should not be emphasized even if taught at all. He should be taught to read or write numbers as he is taught to read or write words. Thus, while writing 36 or 128, the teacher should say, "This is thirty-six," or "This is one hundred twenty-eight." The child thus sees that in any two-figured number as 48 the 4 suggests the "forty" and the 8, "eight," and he can readily read any number of two figures. In the same way, he sees what each figure suggests in a number of three figures.

In reading a number of three or more figures, do not allow the use of "and." Thus, 246 should be read "two hundred forty-six." The use of "and" should be reserved for "mixed numbers" as $346\frac{48}{100}$ or $5\frac{1}{2}$. These are read "three hundred forty-six *and* forty-eight hundredths," and "five *and* two-thirds."

THE YEAR'S ATTAINMENTS

By the end of the year the pupils should have fairly clear number concepts. Through counting they have been able to find the "how many" needed in school, play, or home, thus laying the foundation for their later work. In later grades, the "how many" will be found by a shorter method—addition or multiplication. Aside from definite number concepts they have acquired many size concepts, as large—small, big—little, long—short; many form concepts, as square, circle, rectangle, etc.; many

arrangement concepts, as up—down, over—under, top—bottom, right—left, front—back, etc.; and should be fully equipped to take up the more formal study of number required in the second year.

Besides this, the children have picked up many of the primary facts of addition and subtraction through finding the “how many” that they have needed in their various activities in which number is needed. But no attempt has been made to teach any particular set of facts, nor to drill upon them to develop any skill in recalling them. Their need of number has not yet been great enough to make such work necessary.

CHAPTER III

THE WORK OF THE SECOND YEAR

Number work is now given a definite place on the program. By far the most important work of the second year is the development, drill, and use of the 45 primary facts of addition, and the corresponding 81 subtraction facts. Besides this, a beginning is made with drills that lead to skill in written work, such as adding by endings, and adding single columns. Very little, if any, written work is given in either addition or subtraction. Some, however, may be advisable.

There are certain measures that the child should recognize at sight, and he should be able to estimate lengths, capacities, weights, and time with some degree of accuracy.

He should be able to read numbers to 1000; to read and write dollars and cents, as \$15.65; to read Roman numerals to XII, or farther if need of it arises.

THE COURSE IN OUTLINE

1. The forty-five primary facts of addition.
2. The eighty-one primary facts of subtraction.
3. Preparation for written addition.

(a) Adding single columns of three or four numbers within the primary facts.

(b) Adding by endings in order to add a two-figured number and a one-figured number.

(c) Adding single columns of three or four numbers, using the skill developed under (b).

4. Written addition of two-figured numbers of not more than four numbers. (No carrying, and to be treated but lightly if at all.)

5. Written subtraction of two- and three-figured numbers. (No "borrowing," and to be treated lightly if at all.)

6. The meaning of the words *add*, *subtract*, *sum*, *difference*, and *remainder*.

7. The signs $+$, $-$, and $=$; also $\$$ and $\¢$.

8. Problems including the facts learned.

9. Reading and writing United States money in dollars and cents.

10. Recognition of the units, *inch*, *foot*, *yard*, *pint*, *quart*, *gallon*.

11. Meaning of the terms *dozen*, *ounce*, *pound*, *minute*, *second*, *hour*, *day*, *week*, and *month*.

12. The Roman numerals to XII used in telling time.

13. Reading and writing numbers to 1000.

TEACHING THE UNITS OF MEASURE

All units of measure taught should be presented objectively and used in real situations. In present-

ing the units of length, first present the units and then have pupils estimate lengths of objects whose lengths they desire to know, and then measure them to see how nearly their estimates were correct.

Let pupils see and lift pound and ounce weights. Then let them estimate the weights of objects about them and by actually weighing them test the accuracy of their estimates.

In getting a notion of time, let them count by seconds from a watch that ticks seconds, or let them stand for a second. In the same way, let them stand for a minute. Then let them stand for a few seconds and estimate the time, some pupil with a watch telling them how nearly they estimated correctly. Also have them estimate the time of some school activity. To get the idea of an hour, let them compare it with the length of a recitation, or with two or more recitations, the length of the noon period, the time to go or come from school, etc.

It is not advisable to have fixed periods for instruction in the units of measure, but they should be taught as actual situations require. In this way, the pupils get a much clearer notion of the common units of measure than when these are made a fixed assignment or lesson.

Likewise, in reading and writing numbers, the work should be brought in as needs of reading or writing numbers arise in other work.

THE USE OF PROJECTS IN ARITHMETIC

Mere abstract drill work without a knowledge of its use in problems would, of course, be useless. As stated in Chapter I, there are three distinct and vital phases in the teaching of all facts and processes—their development; the drill; and their uses.

There are numerical aspects of many of the school projects that give an opportunity to use arithmetic and thus bring out the third phase of teaching. But it is a mistake to think that any large part of elementary number work can be based upon projects. The essential technique involved in a good drill lesson—and a large part of elementary number work is drill—excludes the project as a vital part of arithmetic method. The *need* of drill may be discovered in handling number facts needed in a problem or project, and thus the children may desire to drill; and the problems arising in a project may furnish valuable opportunities to apply facts and processes; but beyond this, the project has no place. Even many of the dramatized activities used to motivate drill, fail as valuable drill lessons.

I recently saw a "demonstration lesson" given in a large teachers' training school that clearly illustrates what I have in mind. The notice of the demonstration stated: "A socialized recitation in arithmetic. Special activity: playing grocery store; teacher's aim: to drill upon the first five tables of multiplication."

The children had tables filled with various types of containers and cartons to represent a grocery store. They had brought prices of certain articles found in the local stores. These prices were written upon the blackboard. Most of the prices ended in 0 or 5. Three of the nine digits did not occur among the prices. Clerks were chosen and the children were told to order from 2 to 5 of any article listed. The teacher then gave up control and the children spent the rest of the period—about thirty minutes—in going up to the counter, ordering certain articles, and paying for them with toy money.

The children seemed to enjoy the period fairly well, at least they were well behaved and orderly. But as a drill lesson "in the first five tables of multiplication" it was a period wasted. I sat where I could hear each order given for groceries. I never heard an order for more than two of any one article. Moreover, the digits 9, 8, and 6 did not occur in any price given on the blackboard. As an "application lesson" to show how and when to use multiplication and subtraction, it might have been considered fairly suitable. How it lacked the elements of a good drill lesson will be seen from the following section.

ELEMENTS OF A DRILL LESSON AND PRINCIPLES GOVERNING IT

There is an essential technique involved in any good drill lesson. The teacher should plan each

drill lesson as carefully as a development lesson. She must know what facts she wants to drill upon and why. Drills made up at random and lacking definite purpose or system are a waste of time. Among the principles governing drill are the following:

1. The work should be thoroughly motivated. Drill exercises without interest and attention are worthless.

2. Drill should include each fact of a series.

3. The most time should be given to the most difficult facts.

4. There must be much repetition with attention.

5. The time span between drills must be very short at first and gradually lengthened as the facts become more permanent.

HOLDING THE ATTENTION

The results accomplished in any kind of drill work depend upon the degree of attention given by the pupil. How to secure a maximum of attention is then the vital problem that confronts the teacher. There are no definite laws or sets of devices that will work perfectly under all conditions. Much depends upon the home life of the pupils and the personality of the teacher. But there are certain suggestions that may prove helpful.

1. When pupils show signs of lagging in attention, the form of the drill should be changed.

2. While some concert work may occasionally be

given, it should not consume any important part of the drill period. For in concert work, a few only are really attending to the drills. The large majority are merely saying whatever the leaders say.

3. Individual drill must be so planned as to keep the attention of the whole class fixed upon the combinations given by the one reciting.

4. Use variety in the forms of drill.

5. Do much of the work with a time limit.

6. Instill a desire in each pupil to excel his former record.

7. Use standard tests and create a desire to do as well as pupils of other schools.

8. Have occasional contests among teams made up from the class.

It is the purpose of this monograph to show many types of drill that aim to secure attention. These are distributed throughout the book following the various developments. If any form given in the book fails to get the attention of your class, do not use it. But it may suggest forms that will hold the attention of your class.

THE USE OF GAMES

Games are given to motivate drill work. That is, to secure and hold attention. The many games given in this book have been tried and have been found very helpful. The elements of a good game are: It must be interesting; it must be so arranged

that even with the control of the particular combinations in the hands of the pupils, all facts of a series will come up; it must not eliminate the slow pupil or the one who has made a mistake; and it must as nearly as possible, require all pupils to make all combinations that occur in the game.

Not all drill, however, can be gotten through games. There is too great a danger that certain combinations will receive more than their needed attention while others will get less attention than they need. Carefully prepared drill charts and flash cards are the only means of assuring proper attention to *all* facts of a series. But the games may furnish a motive for the drill from charts, cards, etc. That is, to play the game with greater skill, the pupils may wish to be more rapid and more accurate in all the number combinations.

THE PRIMARY ADDITION AND SUBTRACTION FACTS

There are forty-five primary addition facts and eighty-one corresponding subtraction facts. That is, there are only forty-five ways in which the nine one-figured numbers can be combined. And since there are two subtraction facts to each addition fact, except the doubles, there are eighty-one subtraction facts. Thus for $3+5=8$, there are $8-3=5$ and $8-5=3$.

GROUPING THE FACTS

While there are many logical and pedagogical groupings of the facts into "units of instruction," the grouping here given has been successfully tried and found to be perhaps the most economical.

The two large groups into which the forty-five addition facts have been divided are: (1) those twenty-five sums that do not exceed ten; and (2) those twenty sums that do exceed ten.

These two larger groups are then subdivided into smaller groups as shown by what follows.

THE FIRST TWENTY-FIVE FACTS

Addition is to save counting. When the child finds the "how many" in 5 and 3 by saying "five, six, seven, eight," he is counting. When through memory he announces the sum by seeing 5 and 3, or hearing "five and three," he is adding. But he should first find the "how many" through counting and then remember it to save counting. It is only by thus finding it that he fully realizes the meaning of the expression that "5 and 3 are 8," or is able to find it if the sum is forgotten, or to check up his memory if he is "not sure." This is what is meant by the "presentation" or "development" of a fact.

ADDING ONE

The first nine of the first twenty-five facts are easily disposed of by having the child fully realize

that adding one is but announcing the next number in the order of counting. Thus, showing any number of objects up to nine, add another object and, as it is done, have the child count it in with the others. Thus holding up three objects, place one more with them saying, "Now how many?" Looking at the number held up, the child says, "three," and as one more is placed with them he says, "four," and so on.

At first the teacher will merely say, as she holds up a group, "and one more," the children giving the answer. Then to acquaint them with the meaning of the terms "add" and "adding," she will say as she places one with the group held, "and adding one," "and one added," or "and when I add one." This work is all done with objects before the figures are shown.

Use various objects and the children themselves in developing the facts. Thus, ask certain children to stand before the class. Ask, "How many are standing?" Ask another to join them. Ask, "How many?"

As the objects are shown, the figures that represent the numbers and their sums are written upon the blackboard and finally written upon cards or charts for permanent drill.

These figures should be written in column form and left before the children all the time except when the answers are removed for drill purposes. However, until the facts are fixed they should first

be reviewed before the sums are erased. That is, the pupil should have before him all day long the following chart:

1	3	6	2	5	9	7	4	8
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{7}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{9}$

These numbers should be written in large clear figures and the children should copy them daily for seat work, both in order to fix the facts in mind and to gain skill in making the figures.

In all drill cards of primary facts, the combination and answer should be written on one side, and on the other side, which is the side for drill, merely the combination should be written. When a pupil gives a wrong answer, the card should be turned to show the right answer at once.

• THE SUBTRACTION FACTS

Whether the children are being prepared for the "addition method" of written subtraction or for the "taking-away method," the subtraction facts should first come as mere addition facts. In this method of presenting them, they are merely the known facts of addition and not new facts. So from a knowledge of the above table, the child will be able to give the missing number when the chart is written thus:

*	*	*	*	*	*	*	*	*
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{9}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{10}$

Now writing the sums above as

2	3	6	7	9	4	8	5	10
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{9}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{10}$

have the children say "1 and 1 are 2," "1 and 2 are 3," "1 and 5 are 6," and so on, as the teacher points to the 1 and then to the space below the line, and then to the number above. This order of pointing is to give the right habit in seeing the subtraction facts. While objects may be used as concrete illustrations, the real development of the fact must not be independent of addition. To get the idea of "taking away," hold the groups of objects, say 6 and 1, and ask how many. Then taking either away, ask how many are left. As a matter of economy in learning, however, all subtraction facts must come from the addition facts, and not as new facts.

A DRILL CHART FOR THE ONES

The following chart should be made for permanent use or placed upon some part of the blackboard where it may remain until every fact can be recalled automatically.

Chart I

1	1	1	1	1	1	1
<u>1</u>	<u>3</u>	<u>5</u>	<u>7</u>	<u>6</u>	<u>9</u>	<u>2</u>
1	1	5	7	6	9	2
<u>8</u>	<u>4</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>
1	*	1	*	1	*	1
<u>*</u>	<u>1</u>	<u>*</u>	<u>1</u>	<u>*</u>	<u>1</u>	<u>*</u>
2	4	6	8	7	10	3
1	*	1	*	1	*	1
<u>*</u>	<u>1</u>	<u>*</u>	<u>1</u>	<u>*</u>	<u>1</u>	<u>*</u>
9	5	8	6	10	3	7
9	5	8	6	10	3	7
<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>

NOTE.—The last row is for subtraction.

THE PROBLEMS THAT USE THE FACTS

To make clear the meaning and use of the facts, the abstract drills upon each group of facts should be supplemented with real and concrete problems involving the facts. The problems should first be made from things present as the pictures on the wall, the children themselves, books, pencils, etc. Then problems should be given about very concrete

situations that are vital to the child's interests, as small purchases.

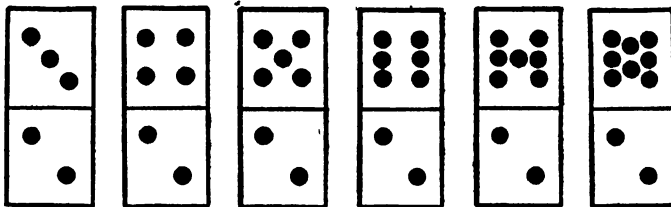
Thus, actually have some number of pupils as 3, or any number to 9, stand; then have another pupil stand, asking, "How many are now standing?" Then without having pupils actually standing, ask questions requiring the children to image the "how many"; as, "If 3 stand in one aisle and 1 in another, how many will be standing?" Also without actually seeing the objects, ask questions involving cents, fruit, nuts, balls, and toys of all kinds.

ADDING TWO

Here, as in all of the first group of twenty-five facts, the child finds the "how many" through counting and remembers it to save counting. The first phase is called the "development" of the fact to make clear its meaning. The second phase, remembering it, comes through drill.

With any group of objects not exceeding 8, add two more and ask how many, the answer coming through counting. Thus if 4 children stand in a row in the front of the room and 2 more join them, have the children begin with "four" and count "five, six," as the 2 join the 4; or holding up a group, as 5, ask how many. Then adding two more (one at a time) ask, "5 and 2 are how many?" having the children begin with "five" and say "six, seven," as they are added.

Using large domino cards, or making them on the blackboard, have the children name the larger number, then count on two more. Thus



looking at a card say "6, seven, eight." The children thus see that adding 2 is counting on two more.

Then write the figures that represent the numbers and their sums and leave them before the children until the facts are fixed in mind and can be recalled without counting.

$$\begin{array}{r} 3 \quad 6 \quad 4 \quad 8 \quad 5 \quad 2 \quad 7 \\ \underline{2} \quad \underline{2} \quad \underline{2} \quad \underline{2} \quad \underline{2} \quad \underline{2} \quad \underline{2} \\ 5 \quad 8 \quad 6 \quad 10 \quad 7 \quad 4 \quad 9 \end{array}$$

For seat work, pupils may make domino cards and find the sums. In making them, let the pupils use the following form:

$$\begin{array}{r} \begin{array}{c} \cdot \cdot \\ \cdot \cdot \end{array} 4 \quad \begin{array}{c} \cdot \cdot \cdot \\ \cdot \cdot \end{array} 5 \quad \begin{array}{c} \cdot \cdot \cdot \cdot \\ \cdot \cdot \end{array} 6 \\ \underline{\quad} \quad \underline{\quad} \quad \underline{\quad} \\ 6 \quad 7 \quad 8 \end{array}$$

And not as so often seen and as shown below:

$$\begin{array}{r} \begin{array}{c} \cdot \cdot \cdot \\ \cdot \cdot \end{array} + \begin{array}{c} \cdot \\ \cdot \end{array} = \begin{array}{c} \cdot \cdot \cdot \cdot \\ \cdot \cdot \end{array} \\ 4 + 2 = 6 \end{array} \quad \begin{array}{r} \begin{array}{c} \cdot \cdot \cdot \cdot \\ \cdot \cdot \end{array} + \begin{array}{c} \cdot \\ \cdot \end{array} = \begin{array}{c} \cdot \cdot \cdot \cdot \cdot \\ \cdot \cdot \end{array} \\ 5 + 2 = 7 \end{array}$$

The child should see the figures written in column form as he is to see them later in written work. The signs $+$ and $=$ are of no value to him now.

This exercise is not "drill" to fix the facts, but "development" to fix the meaning of the written forms. However, care must be used that no wrong figures or sums are written. If such danger exists, the pupils should have the sums written before them and make the cards to correspond with the figures.

DO NOT DEVELOP THE COUNTING HABIT

While the child must first find the facts through counting in order to realize their meanings, and in order that he may find a fact if he has forgotten it, drill must follow until the facts can be recalled without counting. He must see that the advantage of this is to save counting. The first drills should be sight drills from the chart given above in which the sums are written. Have the children spend a few minutes in looking at the figures and their sums, saying "4 and 2 are 6," and so on. You are thus training both the eye and ear. Then covering the sums so they can be shown instantly if an incorrect answer is given or if the child stops to find how many through counting, ask, "How many?" If drill cards are used instead of the chart, or to supplement the chart, have the combinations with answers on one side of the card and show the answer the moment a child hesitates.

The children should copy each group of facts daily from charts showing both the numbers and their sums, but they should not be given the numbers and asked to write the sums as a drill until the teacher is sure that the correct answer can be written without counting. (If the children are given the numbers to add as seat work before the sums are thoroughly known, either they will write a wrong sum that will be fixed in mind and thus delay progress, or they will find the sums through counting and thus form the habit of counting instead of adding.)

Very little work should be done during the second year outside of the class period except in copying results that have been found. That is, copying the numbers and their sums from charts.

Various devices for seat work are offered for sale in which the pupil hunts out the proper numbers and makes up such tables as $4+2=6$. Such work should not be allowed until the teacher is sure that a wrong sum will not be placed. Even then there is but slight value in such work. The drill that the child needs outside of the class period is practice in making the figures. So it will be much more valuable to have the charts copied with the sums given. He is then learning to make the figures, and at the same time getting the correct mental picture of the numbers and their sums in column form as he is to use them later.

ADDING SINGLE COLUMNS OF THREE NUMBERS

As soon as a child knows thoroughly how to add 2, both at sight and orally, he should be taught to add three numbers that involve only known facts. Thus, he is now able to add such columns as the following by adding down.

3	1	3	5	6	4	4	3	6	7	1
1	2	2	2	1	2	2	2	2	1	4
2	2	1	2	2	1	2	2	2	2	2
—	—	—	—	—	—	—	—	—	—	—

In teaching the child to add three numbers, have the columns written on the blackboard and first cover the bottom number with a pad or card until the sum of the first two numbers is called; then remove the card and let the pupil add the third number. Thus in adding the first column above, show 3

1

When the sum is called, remove the card and show the 2, having him add 2 to the 4 that he has already found.

Drill Chart in Addition

Knowing how to add 1 and 2, there are now several combinations of three numbers that come within his knowledge. I have seen such charts made to follow each new table and used with splendid success. The first row gave the table itself. The other drills included these new facts and former facts.

2	3	4	5	6	7	8
<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>
4	5	6	7	8	9	10

1	2	3	4	2	2	2
<u>1</u>	<u>1</u>	<u>2</u>	<u>2</u>	<u>5</u>	<u>6</u>	<u>7</u>
2	2	1	1	1	2	1

2	2	4	4	5	7	8
<u>1</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>2</u>	<u>1</u>	<u>1</u>
1	1	2	2	2	1	1

1	3	3	5	6	7	6
<u>2</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>2</u>	<u>1</u>
2	2	2	2	1	1	2

NOTE.—These columns must be added *down*. To add *up*, combinations not known yet will occur.

THE MISSING-NUMBER DRILLS

For variety and to prepare for the subtraction facts, drills like the following should be given.

*	2	*	2	*	2	*
<u>2</u>	<u>*</u>	<u>2</u>	<u>*</u>	<u>2</u>	<u>*</u>	<u>2</u>
4	6	9	7	10	8	5

In looking at these, the pupil should say "2 and 2 are 4," "2 and 4 are 6," "2 and 7 are 9," and so on.

A MISSING-NUMBER GAME

Using drill cards containing the combinations learned, let one pupil stand before the class and, as

she looks at a card, as for example $\begin{array}{c} 2 \\ 7 \end{array}$

let her say, "I am looking at two numbers that make 9. One is 2, what is the other?" The first to answer "7" gets the card. The game is to see who can get the most cards. The one winning the game then stands before the class and asks the questions.

If the class is not fairly well graded as to skill, the game should be varied so as to give all a chance to recite. Thus the class may choose sides and form two teams. The teacher may ask the questions, alternating from one side to the other, having the pupils answer in turn. Each correct answer wins a card for that team. Those missing are shown the answer to the card missed so as not to miss it again. But the team loses the card which is put back with those not given out. The side getting the most cards wins the game. No game should eliminate the slow or poor pupil. He is the one needing most drill.

PLAYING BLIND MAN

The same drill, as well as drill in the direct addition, can be got from the following game. Each pupil has a large number from 1 to 8 pinned on in full view of the class. One pupil, the blind man, stands in the center of a ring formed by the others,

THE USE OF CHART II

Chart II contains the facts that the child should learn in connection with adding 2. He should become just as familiar with the numbers written in one order as in the other. Thus he should call

$$\begin{array}{c} 2 \\ 5 \end{array} \text{ as readily as } \begin{array}{c} 5 \\ 2 \end{array}$$

The last row of each chart is for subtraction. That the pupil may get the subtraction facts from his addition facts, proceed as follows: When looking at 9, the pupil should say "2 and 7 are 9" as the teacher touches the 2, then the space below the line, and then the 9. This is the "development," or "presentation," stage. As soon as the pupil is taught to think in this order, he gives the answer only. He may also say "9 is 2 and 7" and "9 less 2 is 7," all from the addition fact.

THE GAME OF TAG

To hold the attention of the entire class when having an individual pupil give as many of the combinations on the chart as possible, have the class call out "tag" when a wrong answer is given. The one tagged then sits down and the one first to tag him, or any one that the teacher may choose, recites. The one tagged reënters when he tags some one reciting.

The pupils are anxious to recite, for they want to see how long they can answer without being tagged.

Each child tagged must be shown the right answer to the one missed. When his turn comes to recite again, he begins with the one missed and sees how far he can go without being tagged a second time.

If the teacher wants a pupil to recite until three errors are made, she can change the game into O-U-T Spells Out. The class calls O at the first error, U at the second, and T at the third, then says "O-U-T spells out." However, when a pupil makes many errors in a drill lesson, he is not ready for the drill, and the time spent is wasted. He should be given easier work.

FLASH CARDS

Cards about 4" x 7" should be made, including all of the primary combinations as learned, and these should be used to supplement the chart. They not only give variety, but the order can be changed, and thus they become a better test of ability than the chart does. The cards, too, can be used in many ways and in many games such as the missing-number game given on a preceding page.

The figures on these flash cards should be clearly written in large figures, or should be taken from large calendars. Or, at small expense, the figures can be obtained from office- and school-supply houses already mutilated for pasting.

One side of the card, made for the drill, should contain the two numbers to be added. The other

side should contain the two numbers and their sum. If an error is made, turn the card to show the right answer at once.

THE PROBLEMS

It must not be forgotten that the problems that supplement and follow the abstract drills are just as important as the drills themselves. The problems, however, must answer some real need of the pupil and lead him to the habit of using the facts. This can be done by little problems of his savings, spendings, pets, and toys, as well as from his play, games, gifts, parties, etc. He now has enough facts to answer questions involving three numbers if care is taken that the combinations taken in order do not require combinations that he has not had. A more complete discussion of the nature of a problem will be taken up later. The problems are for the present of the most simple type.

THE PLAYING-STORE CHART

A very large variety of charts can be made for playing store by taking pictures from newspapers, magazines, and catalogs. Thus, from the colored pictures now found in many of the advertisements in our magazines, a large variety of very attractive charts can be made. Thus "The Bakery," "The Lunch Room," "The Grocery," "The Meat Market," etc., can all be represented by colored pictures

taken from current magazines. From illustrated seed and nursery catalogs charts for "The Vegetable Market" and "The Fruit Store" can be made. There are plenty of pictures in black and white to make almost any kind of "Toy Store," "Clothing Store," "Furniture Store," etc.

The prices of the articles should be easily removed so as to vary them to meet the new facts as they are learned.

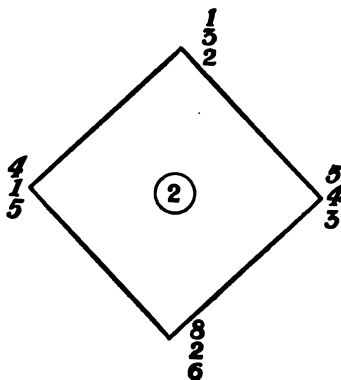
USING THE PLAYING-STORE CHARTS

At first, the teacher may use the chart as a source of problem material, asking the cost of two or more articles, or the difference in cost between any two. Later when the pupils have enough facts so that less care has to be used to keep within the known facts, a pupil may ask the questions. Or, he may be the clerk and take orders for goods and give the total cost of an order. When the pupils are sufficiently developed for it, they may make purchases and pay their bills with toy money and make change. This work, however, is more largely for application than for drill. With the control in the pupil's hands, not *all* facts of the series may be used.

MAKE-BELIEVE GAMES: BASEBALL

By putting the name of some well-known game to a drill device, the children enjoy it much more than a mere drill without particular motive. In any

community where children take an interest in baseball, the game may be played as follows: Make a large baseball diamond about 30 inches square upon the black board. Put the numbers wanted in the drill at the "bases," three or four numbers at each "base." Put the number to be added in the center as the pitcher. Divide the class into two teams. The teacher begins at the top and, going around the diamond, points to any number at a base. The pupil "at the bat" adds 2 (or whatever number is written in the pitcher's place). While one team is "at the bat" all of the other team watch for errors.



If an error is made, all of the opposite team may call out, "Out on first," "Out on second," etc., as the case requires. If no error is made, they call, "Home run," and it is scored for the side playing. This can be made a very interesting game and varied to meet the needs of any of the primary drills.

ADDING THREE

Since the pupil knows $3+1$, and $3+2$ from his former work, there are but *five* new facts in this

group. They are $3+3$, $3+4$, $3+5$, $3+6$, and $3+7$.

These should be presented objectively as in the former work. That is, the pupil should be shown the larger number and then add three by counting. Thus, to find $6+3$, the child, as six objects are first shown and three more placed with them, should begin with 6 and count "six, seven, eight, nine." As the sum is found through counting, the figures representing the numbers and their sums should be written down and remain upon the blackboard or made into a chart until memorized, as in former tables.

Chart III

<u>4</u>	<u>6</u>	<u>3</u>	<u>5</u>	<u>7</u>	<u>1</u>	<u>2</u>
3	3	3	3	3	3	3
<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>
3	3	3	3	3	3	3
<u>4</u>	<u>6</u>	<u>3</u>	<u>5</u>	<u>7</u>	<u>1</u>	<u>2</u>
3	3	3	3	3	3	3
<u>2</u>	<u>1</u>	<u>3</u>	<u>2</u>	<u>4</u>	<u>1</u>	<u>2</u>
2	2	1	1	3	2	1
<u>3</u>	<u>6</u>	<u>2</u>	<u>5</u>	<u>3</u>	<u>1</u>	<u>2</u>
3	6	2	5	3	1	2
<u>*</u>	<u>3</u>	<u>*</u>	<u>3</u>	<u>*</u>	<u>3</u>	<u>*</u>
3	*	3	*	3	*	3
<u>7</u>	<u>9</u>	<u>6</u>	<u>8</u>	<u>10</u>	<u>4</u>	<u>5</u>
7	9	6	8	10	4	5
<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>
3	3	3	3	3	3	3

USING THE CHART

Use the chart as suggested under the preceding charts. The chart does not offer nearly all the possibilities in adding three numbers. In making up supplementary work, be sure that the sum of the first two numbers adding down does not exceed seven, and that one of the two numbers is either 1, 2, or 3.

Supplementary Chart in Adding 3

This corresponds to the chart used when adding 2. It gives the new facts at the top and makes use of any of the old facts with or without the new.

3	4	5	6	7	
<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	
6	7	8	9	10	
2	2	4	2	2	6
1	2	1	3	4	1
<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>
2	1	5	5	4	6
3	3	1	2	3	3
<u>2</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>2</u>	<u>1</u>
3	3	4	3	5	6
2	3	3	4	3	2
<u>3</u>	<u>2</u>	<u>2</u>	<u>3</u>	<u>1</u>	<u>2</u>
3	4	3	5	1	5
3	2	3	3	4	2
<u>1</u>	<u>3</u>	<u>3</u>	<u>2</u>	<u>2</u>	<u>2</u>

NOTE.—To avoid new combinations, add down.

FOUR REMAINING FACTS

There are but four remaining facts of the first group of twenty-five. They are $4+4$, $4+6$, $4+5$, and $5+5$. They are taught in the same general way through counting. However, through their spending, the children know that $5¢+5¢=10¢$ and the other three combinations may be associated with this. (As in all cases, let the pupils see the sums written until the facts become fixed.

Thus, the following table should be kept before the class until the facts are known.

4	4	4	5
4	6	5	5
<hr/>	<hr/>	<hr/>	<hr/>
8	10	9	10

Chart IV

4	6	5	5	4	4	4
4	4	4	5	3	2	1
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
4	4	4	5	3	2	1
4	6	5	5	4	4	4
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
2	1	3	5	2	4	1
2	5	1	3	2	1	3
4	4	5	2	3	1	1
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
*	4	*	5	*	4	*
4	*	4	*	4	*	4
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
8	10	9	10	7	6	5
8	10	9	10	7	6	5
4	4	4	5	4	4	4
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>

ADDING ZERO

Nothing has been said about adding 0. To the child, it presents a real difficulty unless presented as carefully as the other facts. He should read the symbol 0, "zero," and understand that it means "not any." It should not be presented earlier in the work of the combinations, and perhaps it is yet too early. He has not met a need of it yet. To ask, "3 balls and 0 balls are how many?" seems foolish to him. If he has played a scoring game where he has two chances and makes 3 one score and 0 the next, he sees that his score is still 3. In such a way as this, he sees the meaning of the following combinations.

0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0
$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{5}$	$\overline{6}$	$\overline{7}$	$\overline{8}$	$\overline{9}$

These combinations, when presented in connection with scoring, are easily understood and very easily remembered. In the same way (with reference to scoring) use 0 in columns of three numbers as 3, 0, and 5, etc.

A Complete Addition Chart of Former Tables

At this time every pupil should be able to call any combination in the following chart without counting. The chart includes all combinations of the first group, including the nine combinations with zero, written in reverse orders.

1	0	4	0	6	0	0	7	0	3	9	5	2
0	3	0	0	0	8	1	0	5	0	0	0	0
—	—	—	—	—	—	—	—	—	—	—	—	—
0	8	0	0	0	0	1	1	1	4	1	7	6
4	0	6	9	2	7	1	2	8	1	5	1	1
—	—	—	—	—	—	—	—	—	—	—	—	—
1	2	1	1	1	2	3	1	8	4	3	5	5
3	2	9	6	7	1	3	4	1	4	1	5	1
—	—	—	—	—	—	—	—	—	—	—	—	—
9	2	3	2	4	3	8	2	6	7	4	3	5
1	3	4	6	2	5	2	5	3	2	3	2	2
—	—	—	—	—	—	—	—	—	—	—	—	—
7	2	2	5	6	3	2	3	4	4	6	5	6
3	4	7	3	2	6	8	7	6	5	4	4	3
—	—	—	—	—	—	—	—	—	—	—	—	—

NOTE.—These are 64 possible arrangements. They can be made upon a chart of 8 columns by 8 rows. Here they are in 13 columns of 5 rows each. To fill the space, $6 + 3$ is used twice.

REVIEWING THE FIRST TWENTY-FIVE FACTS

The first group of facts usually constitutes the work of the first semester. Both the charts and the flash cards that have been suggested should be used daily. Almost any game that the children play can be made into a number game. Their school and home activities furnish sources for problem material. For those entering the grade in the fall, these facts will be studied during Hallowe'en, Thanksgiving, and Christmas time. Preparing for Hallowe'en parties, buying or making Christmas presents, and trimming

Christmas trees will furnish live activities for problems.

PUSSY WANTS A CORNER

Besides the games already suggested, there are many others that the children can now play. To play Pussy Wants a Corner, two children each have the same number from 2 to 10. This will allow 18 children to play. The numbers are pinned on the children in plain sight. All but one form a large open ring about "pussy," who is in the center. Pussy goes to any one in the ring saying, "Pussy wants a corner." The child to whom it is said asks, "What corner?" Then pussy gives any combination that will make some sum from 2 to 10. If she says "2 and 7," the two 9's exchange places and she tries to get one of the places while the exchange is being made.

To make the game more difficult, and in order to vary it, the one in the center may name any number up to 10. Then a pupil may exchange places with any other pupil whose number with his own makes the number called. Thus, if 8 is called, 7 and 1, 6 and 2, 5 and 3, and 4 and 4 may all exchange.

This makes a good out-of-school game as well as a classroom game. For a classroom game it may be turned into a make-believe game. Thus the teacher may say, "Who changes when I call —?" either naming two numbers and the pupils giving the sum,

or naming the sum and having the pupils give any two numbers that make it.

MAY I TAKE THE PLACE OF — ?

Another little game with the drill cards, somewhat of the same nature, but requiring much less moving about, is enjoyed by most children. It is played as follows. The children stand in line, or in a circle. Each child has a drill card which he holds in full view of all. One pupil out of the line or circle looks at a pupil with a card, say 3 and 7, and says, "May I take the place of 10?" The child asked then comes out of the line or circle and asks a similar question. If, however, the one looking at 3 and 7 asks for any place except 10, the children all clap hands and say, "3 and 7 are 10," and the same pupil looks at another card and asks for a place. The children should exchange cards very frequently in order to get greater drill.

STAGECOACH

The class is divided into two equal sections facing each other, and seated in chairs. One or more on each side is given the same number from 2 to 10 (later to 18). The teacher shows a flash card of addition. Pupils holding the sum of the numbers shown stand, whirl once around, give the sum, and are reseated. The one reseated first wins a point for that section. At frequent intervals the teacher calls,

"Stagecoach," and all pupils change cards in some order as directed by the teacher.

HAVE YOU SEEN MY SHEEP?

An outdoor game called, "Have You Seen My Sheep?" is greatly enjoyed by children. It involves both addition and subtraction and is played as follows.

Each child has a large number from 1 to 10 pinned or hung in full view. All but one, the farmer, form a ring as in playing Drop the Handkerchief. The farmer goes to a pupil and says, "Charles, have you seen my sheep?" Charles, whose number is 6, says, "No, how old is she?" The farmer says, "3 years older than you" (or any number that with 6 does not make more than 10). Charles then adds 3 to his number 6, and looks for 9. If he can catch 9 before she can run around the circle and get back to her place, he may become the farmer. If not, the farmer goes to another and asks a question. He may say, "Martha, have you seen my sheep?" Martha, whose number is 8, says, "No, how old is she?" The farmer says, "Two years younger than you." Martha then subtracts 2 from 8 and looks for 6, whom she must chase.

THE MEANING OF SUBTRACTION

While all subtraction facts should come from the addition facts, whatever method is to be used in

written work, the problems that make clear the meaning of subtraction should bring out the three uses: the amount to be added; the difference; and the remainder. Thus,

1. Mary wants to buy a paper doll that costs 10¢. She has 7¢; how much more money does she need?
2. Helen priced two paper dolls. One cost 10¢ and the other 7¢. How much more did the more expensive one cost?
3. Dorothy had 10¢ and spent 7¢ of it for a paper doll. How much money had she left?

These three problems illustrate the three types of problems that bring out the three meanings of subtraction. But the answers of all come from the fact that 7¢ and 3¢ are 10¢.

The natural answer to the first problem is, "Three cents, for 7¢ and 3¢ are 10¢."

The second answer is, "The more expensive one cost 3¢ more, for 7¢ and 3¢ are 10¢."

And, likewise, the answer to the third is, "She would have 3¢ left, for 7¢ and 3¢ are 10¢."

Advocates of the "taking-away method" of written subtraction often object to the "addition method," thinking that through its use the child does not get the use or meaning of the remainder and difference ideas of subtraction, arguing that subtraction really means a taking away to find a remainder. But it is not the method of finding the

remainder that leads to its real meaning and use. The problems show the meaning of any process and the method of computation is merely the way to find the answer. All such terms as 2 from 9, 9 less 2, remainder, difference, etc., used with the "taking away" method should still be used in connection with problems.

THE LANGUAGE AND CONTENT OF PROBLEMS

The problem material should come from some personal, school, or home issue of interest to the child. The problems should be both concrete and real. That is, they should picture situations very concrete and familiar, and should be questions that would naturally arise because their answers are needed by the one asking them. Too often problems are made up merely to require a process, with no thought as to whether they are real or not. When the answer to a question had to be known in order to formulate the problem, the problem meets no real need and does not lead to the habit of using arithmetic.

They should be given in story form instead of merely stating a process to be performed. Thus, "Yesterday John had 7 marbles. Today he can find but 5 of them. How many has he lost?" This is a real problem in the sense that it may answer a need. The problem, "Yesterday John had 7 marbles but lost 2 of them. How many has he left?" is unreal,

for he must know how many he has left in order to know how many he has lost.

While in the lower grades problems are given mainly to bring out the meaning and the uses of the processes and facts, they should at the same time give as much drill as possible in using the facts. Hence they should make use of as wide a range of number combinations as possible, instead of using the same combinations over and over.

ABILITY TO READ A PROBLEM

Much of the failure to solve a problem in arithmetic is due to poor reading and the inability to understand the problem. That is, the pupil fails to get a clear-cut picture of the situation. When a pupil reads a problem, he often seems to be merely pronouncing empty words. This seems to be much more true of arithmetic than in the reading of a story, or narrative material. There are probably two reasons for this: (1) the pupil is more interested in the numbers involved and the answer than in the situation described; and (2) the problems of most textbooks are stripped of all interesting details and only the bare statement of the facts remains.

If our problems were clothed in story form, describing the situation in greater detail, the pupil would read them with greater interest and get a much more real and vivid realization of the situation. The problems given in this monograph are to

show such types. Thus instead of saying, "Eleven rabbits less 7 rabbits are how many rabbits?" the same problem could be clothed in a more attractive form; thus, "John had 11 little rabbits. When he went to feed them he could find but 7. How many were gone?"

Even this type of problem may tell too little to arouse the same interest and create the same vivid situation as the narrative material of other subjects. Problems all grouped about a single social activity, as "A Hallowe'en Party," "A Thanksgiving Dinner," "A Nutting Party," etc., suggested elsewhere in this book, are perhaps more suitable in getting better reading of a problem in arithmetic. Teachers should strive to get the same quality of reading and understanding in arithmetic as in other studies, and should require as good English in "explaining" a problem as in answering a question in geography or history.

Suggestions as to Types of Real Problems

1. Mary bought three paper dolls. One cost her 6¢, one cost 2¢, and the other 1¢. How much did she spend for all three?
2. John had 9 little rabbits. When he went to feed them he could find but 7. How many were gone?
3. Helen bought 10 pieces of candy and has eaten all but 4 of them. How many has she eaten?
4. Frank wants a top costing 10¢ and now has 6¢.

How much more money does he need in order to buy the top?

5. Dorothy asked 8 little girls to her birthday party. Only 6 of them came. How many did not come?

6. Some children wanted 9 pumpkins for a Hallowe'en party. The boys promised to get 6 if the girls would get the rest. How many did the girls have to get?

7. Helen went to the store for her mother and bought a box of crackers costing 8¢. How much change should she get back from 10¢?

8. To trim a large Christmas tree, the girls brought 4 dozen candles and the boys brought 3 dozen. How many dozen did they all bring?

9. Mary helped her mother set the table for a Thanksgiving dinner. There are 5 in the family and they are to have 3 guests. They must set the table for how many?

10. Frank went to gather the eggs. He found 5 eggs in one nest and 4 in another. How many were in both?

11. Frank spent 6¢ for an ice cream cone and 4¢ for candy. How much did he spend for both?

12. Helen made 8 paper dolls yesterday and today she can find but 5 of them. How many has she lost?

13. Five boys and 4 girls went on a coasting party. How many in the party?

14. Frank wants a ball that costs 10¢. He now has 7¢. How much more does he need to buy it?

15. Helen and Dorothy had a dolls' party. Helen brought 5 dolls and Dorothy brought 4. How many dolls at the party?

16. Frank had 9¢ and spent 6¢ of it for candy. How much had he left?

17. Helen had 7 paper dolls and her sister made her 2 more. How many had she then?

18. Frank and his brother have 8 rabbits. If 5 of them belong to Frank, how many belong to his brother?

19. Ruth spent 3¢ for candy and 6¢ for ice cream. How much did she spend for both?

20. Walter had 10 rabbits and sold 6 of them to Henry. How many had he left?

ADDING AND SUBTRACTING TWO-FIGURED NUMBERS

As soon as the pupils have had the first twenty-five facts of addition they are able to add and subtract two- or three-figured numbers, as long as the numbers are so selected that there is no carrying. The children have but little need of larger numbers yet, although in their buying and spending, and in other school and home interests, they do have a limited need of larger numbers.

There are two possible future values from bringing in a limited amount of written work at this time. It varies the work and thus gives increased interest in the drill, for the written work merely makes use of the former oral drills; and a second value is that the pupils will have formed the habit of working with two or more columns before the difficulty of "carrying" is met.

The "habit" and not the "reason" for adding or subtracting a column at a time or of beginning with the right hand column is the important thing.

Although "habituation" and not "rationalization" is of chief importance in the written work, interest is gained through some real and concrete problem answering a need of the pupils. So a problem involving money may be made the basis of the first written work. If the pupils have first been shown that 15¢ is equal to 1 dime and 5¢, 25¢ equal to 2 dimes and 5¢, 32¢ equal to 3 dimes and 2¢, and so on, the work is easily rationalized. Thus,

John had 23¢ and his mother gave him 15¢ more for running an errand for her. How much did he then have?

This is the way to find out:

How many dimes and cents had he? How
 23¢ many dimes and cents did he earn? Now look
 15¢ at 3¢ and 5¢ and think 8¢ and write it below.
 38¢ Now look at 2 (dimes) and 1 (dime) and think
 3 (dimes) and write it below.

John had 48¢ in his bank but took out 25¢ to buy a knife. How much had he left?

This is the way we find out:

48¢ 48¢ are worth how many dimes and cents?
 25¢ 25¢ are worth how many dimes and cents?
 23¢ First think 5¢ and 3¢ are 8¢, and write 3¢
 below. Then think 2 (dimes) and 2 (dimes) are
 4 (dimes) and write 2 below.

After the processes are thus presented by the use and thought of money, the pupils may apply the

same method to numbers relating to other things or to abstract numbers. To rationalize with other numbers would require teaching the place-value feature of our notation. That is, that 35 means 3 tens and 5 ones, etc. This does not seem advisable at this time.

Drill Exercises in Addition

1.	2.	3.	4.	5.	6.
31	30	24	23	31	14
24	34	40	30	24	32
12	21	22	24	13	22
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>
7.	8.	9.	10.	11.	12.
42	36	25	16	32	38
34	21	32	32	17	40
21	40	21	40	30	21
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>
13.	14.	15.	16.	17.	18.
22	41	53	27	31	26
35	16	13	12	46	31
12	32	21	50	12	42
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>
19.	20.	21.	22.	23.	24.
32	43	26	32	41	31
14	14	32	34	17	25
21	30	21	41	30	43
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>
25.	26.	27.	28.	29.	30.
42	38	16	15	14	18
16	21	43	31	45	21
30	40	40	42	30	50
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>

Drill Exercises in Subtraction

1.	2.	3.	4.	5.	6.
48	54	67	96	88	75
<u>21</u>	<u>12</u>	<u>32</u>	<u>43</u>	<u>34</u>	<u>31</u>

7.	8.	9.	10.	11.	12.
62	84	94	76	86	78
<u>30</u>	<u>52</u>	<u>33</u>	<u>35</u>	<u>64</u>	<u>56</u>

13.	14.	15.	16.	17.	18.
38	59	96	83	97	76
<u>21</u>	<u>35</u>	<u>74</u>	<u>41</u>	<u>55</u>	<u>21</u>

19.	20.	21.	22.	23.	24.
64	84	69	58	76	83
<u>13</u>	<u>31</u>	<u>18</u>	<u>12</u>	<u>42</u>	<u>21</u>

25.	26.	27.	28.	29.	30.
85	76	98	89	48	94
<u>32</u>	<u>45</u>	<u>67</u>	<u>37</u>	<u>16</u>	<u>32</u>

31.	32.	33.	34.	35.	36.
75	68	92	57	83	39
<u>50</u>	<u>37</u>	<u>82</u>	<u>35</u>	<u>62</u>	<u>18</u>

Problems Involving Addition and Subtraction

1. Mary had 31¢. Her mother gave her 20¢ and her father gave her 25¢. How much had she then?

2. Helen wants some paper dolls that cost 35¢ and some extra hats and dresses for them that cost 21¢. How much will all cost?

3. Frank has 65¢. If he spends 42¢ of it for a knife how much will he have left?

4. Walter wants a catcher's glove that costs 95¢. He has 64¢. How much more money will he need to buy it?

5. There are 38 pupils in the second grade of our school. There are 21 girls. How many boys are there?

6. Helen went to the store for her mother and bought 45¢ worth of coffee, 20¢ worth of sugar, and a box of cakes costing 12¢. How much did all cost?

7. Frank got 65 *Saturday Evening Posts* one week and sold all but 12 of them. How many did he sell?

8. Ralph had 58 marbles. He has lost all but 42 of them. How many has he lost?

9. Helen picked 56 sweet peas. If she takes 25 of them to her teacher, how many can she leave at home for her mother?

10. The second grade class of a school had 4 committees to collect little cakes for a party. One committee collected 20, another 24, another 31, and another 21. How many did all four committees collect?

11. John is going to distribute pencils to each member of the class. There are 21 boys and 18 girls in the class. How many pencils will he need?

12. Ralph and Donald each keep chickens. Ralph has 36 and Donald has 25. How many more has Ralph than Donald?

13. John weighs 64 pounds and Walter weighs 52 pounds. How much more does John weigh than Walter?

14. Mary is 41 inches tall and Helen is 44 inches tall. Who is taller and how much?

15. There are 38 pupils in a class. When Dorothy

went to distribute pencils there were but 32 in the box. How many more will she need?

16. Helen went for an automobile ride with her parents. They rode 41 miles before they had lunch and 38 miles after lunch. How far did they ride?

17. Last month Dorothy's mother bought 42 quarts of milk and this month she bought 45 quarts. How many quarts did she buy in 2 months?

18. James wants a kite costing 48¢. He has only 32¢. How much more money does he need?

19. Helen and her sister went to hunt for wild violets. Helen found 56 and her sister found 87. How many more did her sister find?

20. Frank found 43 walnuts and James found 56. How many more did James find?

THE SECOND GROUP OF THE FORTY-FIVE ADDITION FACTS

Through finding sums by counting and by using the facts in problems, the pupils now have a good notion of the meaning of addition. It is not necessary, then, to continue to develop the sums by counting. In groups of things larger than 10, as those included in the last twenty facts, the objects themselves can in no way make the meaning of addition more clear nor help the memory in retaining the facts. So a method shown here is used instead as it is a better help in remembering the facts. To use the method, however, requires that the pupils know the meaning of "teen"; that is, that 15 means

5 and 10, 16 means 6 and 10, and so on. From the sound of "sixteen—six and ten," "seventeen—seven and ten," etc., the pupils accept and remember the fact. Or, by counting they may find out the meaning for themselves.

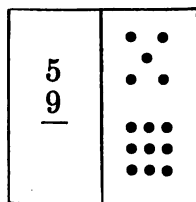
When that is shown, give a few minutes' drill upon calling the following sums, having the pupils look at the two numbers and pronounce the sum in one word:

$$\begin{array}{r} 6 \\ \underline{10} \end{array} \quad \begin{array}{r} 7 \\ \underline{10} \end{array} \quad \begin{array}{r} 8 \\ \underline{10} \end{array} \quad \begin{array}{r} 9 \\ \underline{10} \end{array} \quad \begin{array}{r} 4 \\ \underline{10} \end{array} \quad \begin{array}{r} 3 \\ \underline{10} \end{array} \quad \begin{array}{r} 5 \\ \underline{10} \end{array}$$

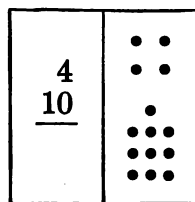
ADDING TO NINE

By the method given here, adding a number to 9 is the simplest of the groups under the last twenty facts. The aim is to show the children that one less than the number added to 9 is the number of *teen*.

To do it, take 9 objects and some other group; then taking one from the second group and putting it with nine, it is the remainder and 10. Thus



is the same as



But 4 and 10 is known by them as fourteen.

PROBLEMS AND GAMES

Problems, playing store, and games should be used as suggested in former combinations, the numbers being changed to bring in these new combinations. As the numbers are larger than those formerly studied, a greater variety of problems and playing-store charts may be used. The problems and games, however, must make use of all former facts learned or they will be forgotten.

Not only should other charts already studied be reviewed very frequently, but all known facts should come up continually in all the applications to games and problems.

ADDING TO EIGHT

In the same general way that adding to 9 was taken up, so can adding to 8 be presented. That is, the pupil can see that it takes 2 of any number added to 8 to make 10 with the 8. So 2 less than the number added is the number of *teen*. Show this clearly, as with 9, then place upon the blackboard or a permanent chart:

$$\begin{array}{r}
 8 \\
 6 \\
 \hline
 14
 \end{array}
 \quad
 \begin{array}{r}
 8 \\
 8 \\
 \hline
 16
 \end{array}
 \quad
 \begin{array}{r}
 8 \\
 4 \\
 \hline
 12
 \end{array}
 \quad
 \begin{array}{r}
 8 \\
 7 \\
 \hline
 15
 \end{array}
 \quad
 \begin{array}{r}
 8 \\
 5 \\
 \hline
 13
 \end{array}
 \quad
 \begin{array}{r}
 8 \\
 3 \\
 \hline
 11
 \end{array}$$

Pointing to 6 and the 4 of 14, 8 and the 6 of 16, 4 and the 2 of 12, 7 and the 5 of 15, and so on, help the pupil remember the sums by seeing that "two less is the number of teen."

Chart VI

8	8	8	8	8	8	8
<u>6</u>	<u>8</u>	<u>4</u>	<u>7</u>	<u>5</u>	<u>3</u>	<u>9</u>
6	8	4	7	5	3	9
<u>8</u>	<u>0</u>	<u>8</u>	<u>8</u>	<u>8</u>	<u>8</u>	<u>8</u>
3	7	6	4	2	2	4
<u>5</u>	<u>1</u>	<u>2</u>	<u>4</u>	<u>3</u>	<u>1</u>	<u>5</u>
6	8	8	7	8	8	8
*	8	*	8	*	8	*
<u>8</u>	<u>*</u>	<u>8</u>	<u>*</u>	<u>8</u>	<u>*</u>	<u>8</u>
14	16	12	15	13	11	17
14	16	12	15	13	11	17
<u>8</u>	<u>8</u>	<u>8</u>	<u>8</u>	<u>8</u>	<u>8</u>	<u>8</u>

Drill upon this chart by the use of problems and games until the facts can be recalled automatically, using the facts also with the other known facts in making up columns of three one-figured numbers. There is a much wider range of numbers for columns of three figures than the ones shown in the chart. All other possible groups should be made up for drill.

THE SIX REMAINING FACTS

But six more primary facts of addition remain. The child may find them through counting and remember the sums, or they can be given him to memorize without development. Finding the sums through counting does not greatly aid the memory

in remembering the facts, but since it gives a final test to see if pupils really know the meaning of addition, they may be asked to find them. In counting, begin with the larger number and count on. Thus to find 7 and 4, starting with 7, say, "Seven, eight, nine, ten, eleven."

Teachers often object to children counting their fingers. As long as they have to count anything, their fingers are as convenient and useful for the purpose as any kind of object. Our number systems were developed by man counting his fingers for centuries. The point is that the facts should be so thoroughly fixed that they are recalled automatically without counting anything.

Chart VII

6	6	6	7	7	7	7
<u>5</u>	<u>6</u>	<u>5</u>	<u>4</u>	<u>7</u>	<u>5</u>	<u>8</u>
6	7	5	4	7	5	8
<u>0</u>	<u>6</u>	<u>6</u>	<u>7</u>	<u>0</u>	<u>7</u>	<u>7</u>
6	7	5	4	7	5	7
2	5	3	2	6	3	2
<u>4</u>	<u>1</u>	<u>3</u>	<u>5</u>	<u>1</u>	<u>4</u>	<u>6</u>
*	7	*	7	*	7	*
6	*	6	*	7	*	8
<u>12</u>	<u>13</u>	<u>11</u>	<u>11</u>	<u>14</u>	<u>12</u>	<u>15</u>
12	13	11	11	14	12	15
<u>6</u>	<u>7</u>	<u>6</u>	<u>7</u>	<u>7</u>	<u>7</u>	<u>8</u>

OUT-OF-SCHOOL GAMES

The pupils have now had all the forty-five primary combinations and the corresponding subtraction facts. Almost any of their regular games can be made into a number game that adds to the interest in the game instead of making it less interesting. The following are examples.

RED ROVER

The game of Red Rover is played in nearly all parts of the country. It is easily made into a number game. Two lines, parallel to each other, and about 60 or 70 feet apart, are marked off. All the children except the catcher stand back of one of the lines. The catcher stands in a circle about midway between the two parallel lines. All children have large distinct numbers from 1 to 9 pinned on them:

The catcher stands in the circle and says, "Red Rover, Red Rover, let — come over," naming any number larger than his own up to 18. The one whose number added to that of the catcher makes the number called then runs across to the other line. If caught, he becomes the catcher, and the catcher takes his place back of the line with the other children. The game is to see if all can finally get across to the other side without being caught. Since, in a large class, several children will have the

same number, all of whom will run when a number is called, some will get safely across.

The out-of-school games can then be turned into a schoolroom drill to furnish a motive for the drill. Thus a "make-believe" Red Rover game can be played as follows: Make two parallel lines on the black board. Back of one line put all but one of the numbers from 1 to 9, each in a circle, and put the remaining number in the center as catcher. Suppose that 5 is in the center. The teacher says, "Let 12 come over." Then she calls upon some pupil to tell who comes over. The pupil says "7." Then the 5 in the center exchanges places with 7, and similar questions are asked.

Both the original game and the make-believe game can be varied. Thus, the pupils may have numbers from 2 to 18 instead of 1 to 9 as above and then when the catcher says, "Let 7 and 5 come over," those having the number 12 will run. This does not give as wide a range of drill as the first method of playing the game, for a pupil will have to keep in mind only the combinations that make his number. Thus one having the number 5 would have to keep in mind only 3 and 2, and 4 and 1.

DROP THE HANDKERCHIEF

In most sections of the country children play Drop the Handkerchief. To make it into a number game, each child would have a number from 1 to 9

pinned on his back, but would know his number. All but one form a ring. One goes around outside the ring. Instead of dropping a handkerchief, he taps the one who is to chase him and says any number that is the sum of the number tapped and a one-figured number. Thus he can tap 6 and say 14. Before 6 can leave her place to chase him, she must say 8, the number which with 6 makes 14. The game is to see if the one who taps can get around to the place left vacant by the one tapped before she catches him. If caught, he goes to the center of the circle and is out of the game.

This game may be used to motivate class drill as a "make-believe" game by putting the nine digits in the form of a circle. Then, as the teacher touches a number, she will give a sum, the pupils telling what made the sum. Thus, if she touches 6 and says 11, the pupils answer 5.

CLASSROOM GAMES

There are many ways of varying the games and drills already suggested. The Missing-Number Game described in a preceding section is always interesting to pupils when drilling upon the primary combinations. This game not only gives drill upon the addition facts, but furnishes a valuable preparation for the subtraction facts at the same time. Another simple form of classroom game is given below.

A GUESSING GAME

The teacher writes upon the blackboard the special facts that she wishes to drill upon. Thus she may write:

7	6	7	9	8	7	9	5	7	9
9	8	7	6	5	5	5	8	6	4
—	—	—	—	—	—	—	—	—	—

One pupil with a pointer stands at the blackboard. Some pupil stands and says, "I am looking at two numbers that make —," naming the sum of some combination written. Thus she may say, "I am looking at two numbers that make 14." The pupil with the pointer may point to 6 and 8, 7 and 7, or 5 and 9. If the pupil points to 7 and 7, the one standing says, "Yes, 7 and 7 make 14, but I am looking at some other two numbers"; and thus make similar replies until the right combination is guessed.

If the pupil at the blackboard points to two numbers that do not make 14, as 6 and 9, the class says, "6 and 9 are 15," and some other pupil takes his place.

This may be varied by having the numbers written on cards. The leader has the cards. A pupil is selected to see if he can "guess" all of them without being caught. The leader, as she looks at 7+5, says, "I am looking at two numbers that make 12. One is 5, what is the other?" And so on through all the cards.

Review Chart

<u>9</u>	<u>7</u>	<u>8</u>	<u>9</u>	<u>7</u>	<u>9</u>	<u>8</u>
<u>8</u>	<u>6</u>	<u>7</u>	<u>6</u>	<u>5</u>	<u>7</u>	<u>6</u>
<u>7</u>	<u>6</u>	<u>5</u>	<u>8</u>	<u>2</u>	<u>4</u>	<u>6</u>
<u>4</u>	<u>5</u>	<u>8</u>	<u>2</u>	<u>9</u>	<u>8</u>	<u>4</u>
<u>6</u>	<u>7</u>	<u>8</u>	<u>6</u>	<u>3</u>	<u>7</u>	<u>7</u>
<u>6</u>	<u>5</u>	<u>8</u>	<u>3</u>	<u>8</u>	<u>3</u>	<u>9</u>
<u>5</u>	<u>6</u>	<u>4</u>	<u>1</u>	<u>7</u>	<u>8</u>	<u>8</u>
<u>9</u>	<u>8</u>	<u>9</u>	<u>7</u>	<u>6</u>	<u>1</u>	<u>9</u>
<u>5</u>	<u>7</u>	<u>9</u>	<u>5</u>	<u>4</u>	<u>2</u>	<u>7</u>
<u>5</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>4</u>	<u>6</u>	<u>7</u>
<u>9</u>	<u>3</u>	<u>6</u>	<u>7</u>	<u>3</u>	<u>4</u>	<u>6</u>
<u>9</u>	<u>5</u>	<u>1</u>	<u>8</u>	<u>9</u>	<u>5</u>	<u>9</u>

NOTE.—Make daily use of the chart and of flash cards. You are more sure of getting *all* combinations than when playing a game. This need take but 5 or 6 minutes daily.

ADDING BY ENDINGS

Before pupils can add a column of numbers that require adding to a number exceeding 10, as $12+5$, they must be drilled on adding a one-figured number to a two-figured number. To do this with skill, they must see that the sums *end* the same as in the forty-five combinations. Thus $12+4$ *ends* in 6, the sum of $2+4$; and $15+8$ *ends* in 3, the same as $5+8$, and so on. The following drill charts give those combina-

tions in which the sum of the right-hand digit of the two-figured and the one-figured number is less than 10.

Specimen Drill

2	12	32	2	12	42
<u>2</u>	<u>2</u>	<u>2</u>	<u>3</u>	<u>3</u>	<u>3</u>
4	14	34	5	15	45
2	12	52	2	12	32
<u>5</u>	<u>5</u>	<u>5</u>	<u>7</u>	<u>7</u>	<u>7</u>
7	17	57	9	19	39
3	13	23	4	14	24
<u>4</u>	<u>4</u>	<u>4</u>	<u>5</u>	<u>5</u>	<u>5</u>
7	17	27	9	19	29
3	13	23	4	14	34
<u>5</u>	<u>5</u>	<u>5</u>	<u>2</u>	<u>2</u>	<u>2</u>
8	18	28	6	16	36

USING THE DRILL CHART

The above drill is but a "specimen," using only 8 of the 20 facts whose sums are less than 10. A chart using all twenty facts should be left before the pupils, showing the sums as above until the pupils fully realize that they all "end" the same as the twenty primary combinations. For drills the numbers are written without the sums as in the following chart.

Drill Chart

1	11	21	1	11	21
1	1	1	3	3	3
—	—	—	—	—	—
1	11	31	1	11	41
2	2	2	7	7	7
—	—	—	—	—	—
1	11	21	2	12	22
6	6	6	3	3	3
—	—	—	—	—	—
2	12	32	2	12	42
2	2	2	5	5	5
—	—	—	—	—	—
1	11	41	1	11	51
4	4	4	5	5	5
—	—	—	—	—	—
2	12	32	1	11	61
4	4	4	8	8	8
—	—	—	—	—	—
4	14	24	2	12	32
5	5	5	6	6	6
—	—	—	—	—	—
2	12	32	3	12	43
7	7	7	3	3	3
—	—	—	—	—	—
3	13	63	3	13	23
4	4	4	5	5	5
—	—	—	—	—	—
3	13	43	4	14	34
6	6	6	4	4	4
—	—	—	—	—	—

ORAL DRILL IN ADDING BY ENDINGS

The purpose of the above drills is very largely to prepare for column addition. But in adding a column, the pupil does not see the two-figured number. Thus in adding the column in the margin, the first sum is 17 to which 6 is added, but the pupil does not see the 17. He merely thinks it. The next sum is 23 to which 5 is added. The next is 28 to which 8 is added. So it becomes clear that while sight drill is necessary it is not sufficient.

To give oral drill, have but one-figured numbers written upon the blackboard and pointing to any one of them, name the two-figured number to be added. Thus pointing to 3 say, "Add 12, add 16, add 23, add 42," etc. And in like manner drill upon all of the combinations given in the chart.

PROBLEMS INVOLVING ADDING BY ENDINGS

Make playing-store charts involving things of interest to the children. Divide each chart into two departments, one in which the price of an article does not exceed 9¢, and the other in which all prices exceed 10¢. Then have pupils give the price of two articles, one from each department. Thus, a "Bakery" chart might include on one side articles above 10¢, and on the other side articles that do not exceed 9¢. Thus,

Bread	15¢	Doughnuts	3¢
Pies	24¢	Cookies	2¢
Cakes	65¢	Rolls	4¢
Cakes	72¢	Cream puffs	5¢
Jelly rolls	58¢	Turnovers	8¢

A lunch chart can be made in the same way in which some serves are more than 10¢ and others less than 10¢. The same can be done with a "Candy Store," "Toy Shop," "Christmas Shopping," etc.

The children enjoy collecting the pictures and prices and making the charts. But since the purpose of using the charts is to furnish drill, the teacher should guide the children in selecting prices that will bring in as large a variety of numbers as possible, instead of the same numbers several times.

THE SECOND ADDING-BY-ENDINGS GROUP

This group involves those combinations whose one-figured number makes with the right-hand figure of the other ten or more. As in the first group, the pupil is to study the endings and see that all *end* as in the primary facts. For this purpose a chart like the one below should remain before the pupils showing the sums. There is no thought of carrying. The pupil may satisfy himself of the correctness of the results by counting, then observe the endings and that the ten's digit is increased by 1.

Specimen Chart

2	12	22	4	14	34
<u>8</u>	<u>8</u>	<u>8</u>	<u>6</u>	<u>6</u>	<u>6</u>
10	20	30	10	20	40

2	12	22	5	15	35
<u>9</u>	<u>9</u>	<u>9</u>	<u>6</u>	<u>6</u>	<u>6</u>
11	21	31	11	21	41

4	14	44	5	15	25
<u>8</u>	<u>8</u>	<u>8</u>	<u>7</u>	<u>7</u>	<u>7</u>
12	22	52	12	22	32

5	15	35	5	15	25
<u>8</u>	<u>8</u>	<u>8</u>	<u>9</u>	<u>9</u>	<u>9</u>
13	23	43	14	24	34

7	17	27	6	16	36
<u>7</u>	<u>7</u>	<u>7</u>	<u>9</u>	<u>9</u>	<u>9</u>
14	24	34	15	25	45

8	18	28	9	19	29
<u>8</u>	<u>8</u>	<u>8</u>	<u>9</u>	<u>9</u>	<u>9</u>
16	26	36	18	28	38

Drill Chart

1	11	31	3	13	43
9	9	9	7	7	7
—	—	—	—	—	—
3	13	23	5	15	25
8	8	8	7	7	7
—	—	—	—	—	—
5	15	35	5	15	35
6	6	6	9	9	9
—	—	—	—	—	—
7	17	37	16	16	46
7	7	7	4	4	4
—	—	—	—	—	—
2	12	32	3	13	23
8	8	8	9	9	9
—	—	—	—	—	—
6	16	46	7	17	57
6	6	6	8	8	8
—	—	—	—	—	—
8	18	38	6	16	46
9	9	9	8	8	8
—	—	—	—	—	—
5	15	45	4	14	34
8	8	8	9	9	9
—	—	—	—	—	—
6	16	36	7	17	27
7	7	7	9	9	9
—	—	—	—	—	—
8	18	38	4	14	54
8	8	8	7	7	7
—	—	—	—	—	—

4	14	54	6	16	36
8	8	8	9	9	9
—	—	—	—	—	—
9	19	29	2	12	32
9	9	9	9	9	9
—	—	—	—	—	—

NOTE.—The above drill contains all but $5+5$, which should also be used, but being the easiest and most common, it was omitted for lack of room.

Throughout the term much drill from this chart should be given, and the use of the facts should be given in problems, if pupils are to add columns with any skill. It is lack of sufficient drill in these facts that causes pupils to add slowly or to count when adding columns.

Specimen Problems that Can Now be Solved

1. John spent 39¢ for a knife and 8¢ for a top. How much did both cost?
2. Helen had 28¢ and earned 7¢ dusting the room. How much had she then?
3. Frank had 54¢ and earned 9¢ more. How much had he then?
4. Mary earned 25¢ one week drying dishes and 8¢ more by dusting the room. How much did she earn in all?
5. Ralph spent 59¢ for a box of candy for his mother and 6¢ for an ice cream cone for himself. How much did he spend for both?
6. After giving his sister 6 pieces from a box of candy, Frank found he had 18 pieces left. How many were in the box?

7. Ralph's mother sent him to the store to get a pound of 48¢ coffee and a 6¢ cake of soap. How much money did both cost?

8. Frank bought a repeating air rifle and a box of wooden balls for it for 67¢, and an extra box of balls for 6¢. How much did both cost?

9. Helen's mother gave her 68¢ to buy a pound of butter and gave her 6¢ extra to get an ice cream cone. How much money did she give her in all?

10. Mrs. Brown bought 87¢ worth of groceries and paid 7¢ extra to have them delivered. How much did she pay in all?

11. Helen paid 18¢ for a paper doll and 9¢ for extra clothing for it. How much did all cost?

12. Walter went to the store for his mother. He bought a dozen oranges for 45¢ and three lemons for 8¢. How much did he pay for all?

13. After selling 9 of his rabbits, Frank had 14 left. How many had he before selling any?

14. Gertrude gathered some roses. After counting out 8 to take to school, she had 14 left for her mother. How many did she gather?

15. Harry had a basket of apples for his Hallowe'en party. After each of 16 children had one, there were 5 apples left. How many were in the basket?

ADDING SINGLE COLUMNS

After considerable drill upon adding a two-figured number and a one-figured number, pupils can begin to add columns in which a sum exceeds 10 before all are added. The work should be well

graded and closely connected with the adding by endings. Thus,

	6	8	3	5		4	8	6
12	6	4	9	7	13	9	5	7
<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>
	5	4	6	9		9	5	7
12	7	8	6	3	13	4	8	6
<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>
	6	4	8	2		6	5	7
11	5	7	3	9	14	8	9	7
<u>8</u>	<u>8</u>	<u>8</u>	<u>8</u>	<u>8</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>

Drill Chart

8	5	2	8	4	8	9	7
4	7	9	3	7	4	4	6
<u>4</u>	<u>5</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>6</u>	<u>4</u>	<u>6</u>
5	7	4	6	8	2	4	6
8	6	9	7	2	9	7	4
<u>5</u>	<u>4</u>	<u>3</u>	<u>3</u>	<u>5</u>	<u>4</u>	<u>5</u>	<u>8</u>
6	7	5	8	7	7	6	8
8	7	9	6	7	8	9	8
<u>4</u>	<u>3</u>	<u>5</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>3</u>
7	8	9	6	5	8	5	4
9	9	8	7	9	6	9	9
<u>2</u>	<u>1</u>	<u>2</u>	<u>6</u>	<u>4</u>	<u>5</u>	<u>3</u>	<u>6</u>

USING THE DRILL CHARTS

Use the last two charts very frequently. They contain a very large variety of combinations. After being used as sight drill in the class, let the pupils copy them and add for seat work. Should a wrong sum be recorded, it does not have the effect of forming a wrong notion as in copying a wrong sum for one of the primary combinations. The pupil should be taught to check his work, however, by adding in opposite directions before recording the sums.

Columns that Use the Second Group of Adding-by-Endings Facts

[illegible]

4	9	6	6	6	8	7	3
8	5	7	6	9	9	9	9
9	9	8	9	6	4	5	9

6	4	6	8	8	6	7	6
7	9	8	4	6	9	7	8
9	8	7	9	7	5	8	9

9	6	9	4	9	7	6	9
9	9	7	9	7	8	9	5
<u>5</u>	<u>8</u>	<u>6</u>	<u>9</u>	<u>6</u>	<u>7</u>	<u>7</u>	<u>8</u>

Drill Chart

6	7	4	3	6	5	5	6
9	8	5	1	3	4	2	3
6	7	7	9	9	7	8	5
3	4	6	9	4	6	7	8
—	—	—	—	—	—	—	—
2	1	6	4	1	2	5	3
4	6	3	3	5	5	3	4
7	7	8	9	8	8	8	8
9	8	5	6	9	8	6	9
—	—	—	—	—	—	—	—
2	6	3	1	7	4	2	3
7	3	5	8	2	5	6	9
9	8	8	9	8	7	7	7
6	7	9	7	9	8	8	3
—	—	—	—	—	—	—	—
8	9	8	9	7	3	2	3
4	7	7	3	4	6	9	6
6	5	6	6	6	8	6	8
4	6	4	6	9	4	7	5
—	—	—	—	—	—	—	—

WRITTEN ADDITION

The written work of the second grade should be optional. The work already outlined is far more important than written work. Skill in adding by endings and in adding single columns must also be acquired before much written work is given. Yet a very limited amount of written work may answer certain needs of the child and help to vary the drill in oral work.

Written addition, when there is no carrying, has already been discussed. In the carrying process, the "habituation" of carrying is far more important than its "rationalization." By beginning with very simple exercises where money is involved, however, the carrying process can easily be shown. Thus,

If John had 18¢ and earned 15¢ more, how much had he?

This is the way to find out:

18¢	18¢ is equal to 1 dime and 8 cents.
<u>15¢</u>	15¢ is equal to 1 dime and 5 cents.
33¢	First, adding the 8¢ and 5¢, the sum is 13¢, which is equal to 1 dime and 3¢. Write the 3¢ under the column of 8¢ and 5¢. Add the 1 dime to the column of 1 dime and 1 dime.

Mary had 24¢ and her mother gave her 18¢ more. How much had she then?

This is the way to find out:

24¢	24¢ is equal to 2 dimes and 4 cents.
<u>18¢</u>	18¢ is equal to 1 dime and 8 cents.
42¢	Adding 4¢ and 8¢, the sum is 12¢ or 1 dime and 2¢. Write the 2¢ under the 4¢ and 8¢, and add the 1 dime to the column of 2 dimes and 1 dime and the sum is 4 dimes.

Continue simple problems like this, explaining each in this simple way, until the process becomes automatic. Then select numbers a little larger in

which there will be 2 to carry, otherwise the pupil will form the habit of carrying but 1. Thus,

Mary went to the store for her mother and bought oranges costing 38¢, lemons costing 27¢, and a pineapple costing 18¢. How much did all cost?

This is the way to find out:

38¢ 38¢ is equal to 3 dimes and 8 cents.

27¢ 27¢ is equal to 2 dimes and 7 cents.

18¢ 18¢ is equal to 1 dime and 8 cents.

83¢ Adding 8¢, 7¢, and 8¢, the sum is 23¢ or 2 dimes and 3¢. Write the 3¢ under the 8, 7, and 8 cents and add the 2 dimes to the column of 3 dimes, 2 dimes, and 1 dime.

Since the child can likely visualize small amounts of money of this size, he sees the reason for the carrying, and grows into the habit of automatically recording the right-hand digit of a sum and carrying the left-hand digit to the next column.

Through simple problems involving money, keeping the sums less than \$1, the pupil gets the proper habit of carrying. Then tell him that whatever kind of number is used, the process is the same, and proceed with abstract drill. This is all the rationalization needed and all that can be understood at this time.

CHECKING WORK

Pupils should develop the habit of checking all computation. That the pupil knows how to get a

correct result is more important than the time it takes to get it, although the matter of speed is important and must be encouraged.

To check addition, the pupil from the first should be required to add each column in both directions and get results that agree before recording them. The first result should be set down on a bit of scrap paper or at one side until the check shows it to be correct.

Drill Exercises

These may either be placed upon the blackboard for the class to copy and add or they may be dictated by the teacher for blackboard or seat work. The answers are given here in the drill to save the time of the teacher in correcting the results.

1.	2.	3.	4.	5.	6.
38	27	54	63	35	42
40	46	20	29	26	27
24	30	39	54	80	30
17	28	45	20	49	18
<u>119</u>	<u>131</u>	<u>158</u>	<u>166</u>	<u>190</u>	<u>117</u>

7.	8.	9.	10.	11.	12.
34	28	56	45	53	61
26	34	36	26	32	38
21	16	20	31	30	40
87	40	19	28	27	17
<u>168</u>	<u>118</u>	<u>131</u>	<u>130</u>	<u>142</u>	<u>156</u>

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13.	14.	15.	16.	17.	18.
18	27	40	51	48	37
46	37	38	16	21	25
20	20	16	30	30	40
35	19	29	29	28	18
<u>119</u>	<u>103</u>	<u>123</u>	<u>126</u>	<u>127</u>	<u>120</u>
19.	20.	21.	22.	23.	24.
35	38	74	52	63	47
26	26	17	17	19	26
19	19	10	30	30	40
30	40	45	28	24	32
<u>110</u>	<u>123</u>	<u>146</u>	<u>127</u>	<u>136</u>	<u>145</u>
25.	26.	27.	28.	29.	30.
52	64	73	58	84	65
16	20	26	16	21	27
20	19	10	35	37	20
39	38	45	10	20	39
<u>127</u>	<u>141</u>	<u>154</u>	<u>119</u>	<u>162</u>	<u>151</u>

THE YEAR'S ATTAINMENTS

There are no established universal standards for the achievements of the second year. But teachers wish to take stock of the year's work and summarize the results. The results of the year might be classified under three heads: *knowledge*, *skill*, and *attitude*.

As to *knowledge*, the pupil should have a complete control of the forty-five primary facts of addition

and the corresponding subtraction facts, being able to give the sum of any of the combinations without counting.

He should know the meaning of addition and subtraction, and also the meaning of the common units of measure.

As to *skill*, he should be able to use the primary facts in simple problems, to add three one-figured numbers, and a two-figured number to a one-figured number with some skill.

Also, he should have developed habits of order and neatness, and an increased power of attention.

As to *attitude*, he should take pride in his knowledge of number; enjoy working with numbers; desire to know more and to work more with them; and see that they are useful to him in many ways.

CHAPTER IV

THE WORK OF THE THIRD YEAR

The most important new work of the third year is the multiplication tables and the corresponding division tables; written multiplication and division by one-figured multipliers and divisors; and the use of the facts in simple problems. The number facts in addition and subtraction must be constantly reviewed and much time spent in written addition and subtraction. The problems should be grouped about social interests of the pupil.

The units of measure needed in this grade are determined by the pupil's needs, which vary in different localities.

THE COURSE IN OUTLINE

1. Frequent reviews of the primary addition and subtraction facts.
2. Frequent reviews in adding by endings and in adding single columns.
3. Reading and writing numbers to 10,000.
4. Written addition limited to four three-figured numbers.
5. Written subtraction limited to four-figured numbers.

6. The multiplication tables and written multiplication limited to multipliers of one-figured and to multiplicands of three-figured numbers.

7. The division tables, and short division limited to quotients of three figures and no remainders.

8. The meaning of multiplier, product, divisor, and quotient.

9. Review of units of measure taught in the second year.

A REVIEW OF FORMER WORK

The pupils will still need much review of the work of the second year. The charts used in that grade should be reviewed very frequently. Adding by endings and column addition in particular should receive at least half of the time of the third year. Use the same type of games, playing-store charts, and types of problems as suggested for the work of the second grade, varying them, of course, to meet the growing powers and maturity of the children.

ADDING DOLLARS AND CENTS

Both to meet the child's needs and to furnish motivated material for problems, the pupil should be taught to read and to write dollars and cents before much written work is taken up. Keep the amounts of money within the pupil's power to image them. Thus,

If Frank spent \$2.35 for a coaster, and \$1.78 for a pair of roller skates, how much did both cost?

This is the way to find out:

\$2.35 Adding as before, 5¢ and 8¢ are 13¢ or 1
1.78 dime and 3¢. Write 3¢ in cents' column and
 \$4.13 add the 1 dime to 3 dimes and 7 dimes,
 making 11 dimes. 11 dimes are equal to \$1
 and 1 dime. Write the 1 dime under dimes' column
 and add the \$1 to \$2 and \$1, making \$4.

This is all the explanation necessary. Drill with similar problems until the recording and carrying become fixed habits.

Problems for Addition

1. Helen's mother bought her for her play room a combination blackboard costing \$4.87 and a little desk and chair costing \$6.98. How much did both cost?

2. Ralph's father bought him a tool chest costing \$3.98 and a "Meccano" set costing \$6.75. How much did both cost?

3. Lucille's mother bought her a toy phonograph costing \$4.47, a set of records and needles costing \$1.35, and a little table costing \$2.48. How much did all cost?

4. Find what our Christmas tree and decorations cost. The tree was 85¢, 4 bells cost 28¢, 20 feet of tinsel cost 68¢, 3 dozen candles 39¢, and 12 colored balls cost 79¢.

5. Find the cost of our Hallowe'en party. The decorations cost \$2.15, the apples cost \$1.75, the doughnuts cost 98¢, and the cider cost \$1.65.

6. Find what the refreshments cost for Helen's party. The ice cream cost \$2.10, the cakes cost 85¢, and the candy and nuts cost \$1.38.

7. Dorothy's mother bought her a doll costing \$3.49, doll furniture costing \$1.19, and a toy china set costing \$2.13. Find the cost of all.

8. Mrs. Brown bought John a coaster costing \$4.98, Helen a doll costing \$3.84, and baby a kiddie horse costing \$2.39. Find the cost of all.

NOTE.—The numbers used in the problems should be so selected as to give valuable drill in combining numbers, as well as power to apply processes.

PROBLEM CHARTS

Beautiful charts of "The Toy Shop," "The Doll Shop," "Wheeled Toys," "Wagons and Coasters," "Mechanical Toys," "Building Toys," "Tool Chests," "What Every Girl Wants," "What Every Boy Wants," "Boys' Clothing Store," "Girls' Clothing Store," and dozens of others, can be made from pictures taken from catalogs, newspapers, and magazines. These can be used in playing store and thus make the work not only much more real but much more interesting.

It seems unbelievable that some teachers and authors still expect these little children, but nine or ten years old, to be interested in finding "how many sheep the farmer had," "how many bushels of wheat Mr. Jones raised," "how many acres a farmer had," and like questions. Yet both teachers and

textbooks make much use of just such material. It would seem that a teacher who can stand before her class and give such work is wholly lacking a sense of humor.

DRILLS IN ADDITION

The pupils need much drill in abstract work in addition in order to develop speed and accuracy. Rivalry between sections, working with a time limit, and other tests of skill that bring out the game spirit in abstract work, are sufficient to keep up interest. Pupils find such work just as interesting as those drills clothed in problem form. In fact, the problems are not given for the primary reason of motivating drill, but to clarify the meaning of the process, to show when to use it, and to lead to the habit of using arithmetic to answer problems of vital personal interest. Yet when problems are given, they may be so selected as to give valuable drill while serving their real purpose.

DRILLS THAT SAVE COPYING NUMBERS

The schools are flooded with various devices for drill in elementary number work that save copying the exercises and require only the writing of the answers. Some of this kind of work is all right, but it should not be the only kind. Pupils need drill in copying figures, both from sight and from dictation, for they need practice in making the figures until

they can be formed neatly without conscious effort. Besides this need, tests show that in all grades pupils make a great many errors in copying numbers either from sight or dictation, and hence that they need much drill along this line. The teacher should understand, then, that in using such devices, she is giving greater time to the recalling of number facts, but in so doing she is neglecting another important phase of the work. Such devices, then, should not be used for the major part of the drill work, unless time be given in drills upon copying numbers from sight and dictation.

Drill Exercises in Addition

(These are to be dictated by the teacher and be copied by the pupil. The answers are given for the teacher's convenience in checking the pupils' papers)

1.	2.	3.	4.	5.	6.
38	54	38	24	82	63
42	38	29	78	36	71
63	76	46	91	74	89
<u>19</u>	<u>91</u>	<u>75</u>	<u>53</u>	<u>59</u>	<u>54</u>
162	259	188	246	251	277

7.	8.	9.	10.	11.	12.
65	59	81	47	37	93
48	48	79	96	96	28
92	63	65	58	45	76
<u>37</u>	<u>72</u>	<u>34</u>	<u>32</u>	<u>28</u>	<u>45</u>
242	242	259	233	206	242

13.	14.	15.	16.	17.	18.
19	38	47	53	84	56
87	79	69	86	57	89
65	46	80	40	90	73
<u>20</u>	<u>50</u>	<u>52</u>	<u>92</u>	<u>36</u>	<u>40</u>
191	213	248	271	267	258
19.	20.	21.	22.	23.	24.
80	97	83	76	38	42
76	80	70	98	92	79
93	36	69	25	70	80
<u>54</u>	<u>45</u>	<u>52</u>	<u>40</u>	<u>65</u>	<u>65</u>
303	258	274	239	265	266

THE ADDITION METHOD OF SUBTRACTION

Since the addition method of written subtraction is rapidly coming to be the method generally used, it is the method given here. Many teachers having used and taught the "taking-away" method find trouble in teaching this method. The reason is that they are yet thinking of "taking away" instead of "adding to" and mix the language of the old method with that of the new. Not only that, but they use much of the old method with the new. Some recent textbooks even do the same. Thus they explain a problem like the following as shown below.

"Since 2 is less than 7, take 1 of the 8, making
 82 1 ten and 2 or 12. 7 and 5 are 12, write 5.
 27 Since 1 of the 8 has been taken, 7 remains.
 55 2 and 5 are 7, write 5."

Thus it is seen that the method shown here is a combination of both the "taking-away" method and the "addition" method. No teacher can hope to teach such a method successfully or to develop much skill when using it. The pupil should be taught to think, "7 and 5 are 12, write 5; 1 to carry and 2 are 3, and 5 are 8, write 5."

REVIEW OF SUBTRACTION FACTS

To prepare for written subtraction where carrying is involved, review those facts whose sum is a two-figured number. In reviewing such facts as

10	11	13	12	14	16	15
<u>7</u>	<u>5</u>	<u>8</u>	<u>6</u>	<u>9</u>	<u>7</u>	<u>8</u>

the pupil is thinking of adding, not "taking from," and says "7 and 3 are 10," "5 and 6 are 11," "8 and 5 are 13," and so on.

Drill Chart

10	15	13	11	14	13	12	16	10
<u>1</u>	<u>6</u>	<u>9</u>	<u>2</u>	<u>9</u>	<u>5</u>	<u>3</u>	<u>7</u>	<u>8</u>
11	13	10	14	12	15	11	17	12
<u>8</u>	<u>4</u>	<u>2</u>	<u>5</u>	<u>8</u>	<u>7</u>	<u>3</u>	<u>8</u>	<u>9</u>
10	14	12	13	10	17	14	13	11
<u>9</u>	<u>8</u>	<u>4</u>	<u>6</u>	<u>3</u>	<u>9</u>	<u>6</u>	<u>7</u>	<u>6</u>
12	10	16	11	14	12	10	15	11
<u>7</u>	<u>6</u>	<u>8</u>	<u>5</u>	<u>7</u>	<u>5</u>	<u>4</u>	<u>8</u>	<u>7</u>
11	13	12	10	15	11	16	10	18
<u>4</u>	<u>8</u>	<u>6</u>	<u>5</u>	<u>9</u>	<u>9</u>	<u>9</u>	<u>7</u>	<u>9</u>

All subtraction, where carrying is involved, depends upon some of these forty-five facts, so the pupil should be drilled upon them daily until the results are given automatically.

THE NEXT PREPARATORY STEP

The pupil has already subtracted such problems as $84 - 21$. Here he thought, "1 and 3 are 4, 2 and 6 are 8."

To solve such a problem as $82 - 37$, he has to reason as follows:

Some number was added to 37 to make 82. Then he sees that 2 cannot be the sum of 7 and any number. Show him from the tables that it is the 2 of the 12, the next number ending in 2.

So he thinks 7 and 5 are 12, just as in the tables. The 2 is the 2 that was recorded of the 12, just as in any addition, and the 1 was carried to the 3, making 4. Then 4 and 4 are 8. The work should then be checked by adding up. Thus, "5 and 7 are 12; write 2, carry 1; 5 and 3 are 8."

Pupils never ask where the 12 comes from. But teachers brought up on the "taking-away" method often do, and they try to explain it from that method. But that is not the explanation. 82 is the *sum* of 37 and some number. Since 7 is larger than 2, the 2 merely shows that something must be added to 7 to make 12, the next number ending in 2. The 1 is

carried because in *any* sum of 12 the 2 is recorded and 1 is carried.

So before any further written work is given, write a number of problems upon the blackboard as

71	80	92	63	82	93	64
<u>26</u>	<u>27</u>	<u>47</u>	<u>25</u>	<u>65</u>	<u>48</u>	<u>37</u>

Covering all but the right hand column, ask "Now what must we think?" The answer is, "Think 71 11, carry 1." For the next, the answer is, 26 "Think 10, carry 1," "Think 12, carry 1," and so on. Since the sum of two one-figured numbers can never be more than 18, there is never more than 1 to carry. Give daily drill of this kind until pupils can tell at once what to "think." That is, the pupil should get the habit of thinking 10, 11, 12, 13, . . . , or 18, as he sees the number above smaller than the one below it, and of carrying 1 under all such cases.

PROBLEMS IN SUBTRACTION

The meaning and use of a process is brought out by the problems and not by the method of performing the written process. There are three general types of problems all answered by subtraction. They are: (1) to find what must be added to a number to make a certain sum; (2) to find what is left when a certain amount has been taken away from a number; and (3) to find the

difference between two numbers. That is, the problems involve the addition idea, the remainder idea, and the difference idea. These three types are shown in the following problems.

FINDING WHAT TO ADD

Frank wants a catcher's glove that costs 95¢. He now has 58¢. How much more does he need?

This is the way we find out:

Write the larger number above and the smaller one below it. Think, "8¢ and 7¢ make 15¢."
 95¢
 58¢ Write 7¢ and carry 1. Think, "1 to carry and 5
 37¢ make 6, and 3 make 9." Write 3. Check by adding up. 7¢ and 8¢ are 15¢; 5¢ is written and 1 dime carried. 1 and 3 and 5 are 9.

FINDING A REMAINDER BY SUBTRACTION

Frank had 95¢ and spent 58¢ of it for a ball. How much had he left?

This is the way we find out:

Write the larger number above and the smaller one below it. Think, "8¢ spent and 7¢ left
 95¢ will make 15¢ that he had." Write 7¢, carry 1.
 58¢ Think, "1 to carry and 5 and 3 make 9."
 37¢ Write 3. Check as above. Hence 37¢ are left, for 37¢ and 58¢ make the 95¢ that he had.

FINDING A DIFFERENCE BY SUBTRACTION

Frank spent 95¢ for a glove and 58¢ for a ball.
How much more did the glove cost than the ball?

This is the way we find out:

Write the larger number above and the 95¢ smaller one below it. Think, "8¢ and 7¢ are 58¢ 15¢." Write 7¢. Think, "1 to carry and 5 and 3 37¢ are 9." Check as before. So the difference is 37¢, for it would take 37¢ more than 58¢ to make 95¢.

This should meet the objection of teachers that by the addition method of subtraction the pupils fail to see and use the remainder and difference ideas and language of subtraction. The method makes no difference in the use or thought of subtraction. The problems must be so varied and expressed as to bring out all three meanings.

Written Problems in Addition and Subtraction

The pupil is now ready for problems not classified as to process. He should now have problems in which he must determine for himself whether to add or subtract. But to do this, the problems must all be about affairs very concrete to him.

The problems in subtraction necessarily involve but two numbers. So if *all* addition problems given involve three or more numbers, the pupil may judge from the number of numbers involved what to do,

instead of really seeing from the nature of the problem what process is to be used. Hence, for the purpose of teaching the meaning of addition and subtraction and what process to use, many of the addition problems should involve but two numbers.

While the primary use of the problems is to bring out the meaning and use of the processes, they should at the same time give valuable drill in the various combinations and an appreciation of the need of arithmetic.

1. James looked at two different sizes of the "Gilbert Wheel Toy." The price of the small one was \$5.47 and of the large one \$8.95. How much more does the large one cost?

2. Frank wants to buy a large-size "Gilbert Wheel Toy" costing \$8.95. He has \$6.38. How much more money does he need?

3. Mary's father bought her little four-year-old brother a hobby-horse costing \$5.67 and a "Two-in-One" set of wagon blocks costing \$1.48. How much did both cost?

4. How much more did the hobby-horse cost than the wagon blocks?

5. Helen had \$2.35. She bought her little brother a train of cars for \$1.48. How much had she left?

6. Mr. Brown bought his little boy a "Kiddie Racer" for \$1.47 and a hobby-horse for \$1.79. How much did both cost?

7. Mary's mother bought her a doll for Christmas. She looked at a doll 20 inches high that cost \$6.25, and

the same kind of doll but 18 inches high that cost \$5.38. How much less did the 18-inch doll cost?

8. How much would both have cost?

9. Dorothy saw a doll costing \$2.18 that she wanted to buy. She has \$1.35 in her bank. How much more will she need?

10. Helen's mother bought her a furniture set for her dolls costing \$2.47, and a hobby-horse for her little brother costing \$3.28. How much did both cost?

11. For Christmas Mary's mother bought her a doll costing \$2.98 and a doll trunk costing \$1.19. How much did both cost?

12. Walter has \$5.35 in his bank. He wants to buy a "Gilbert Erector" costing \$2.19. If he buys it, how much money will he have left?

13. Helen bought her little sister a doll costing \$1.48 and a set of dolls' dishes costing 67¢. How much did both cost?

14. Helen looked at two toy china tea sets. One cost \$2.47 and the other \$1.98. How much can she save by buying the cheaper set?

15. Frank wants a tool set. He can get a set for \$3.50 or a cheaper one for \$2.78. How much can he save by getting the cheaper set?

16. John is saving money to buy an "Erector" costing \$2.19 and a tool chest costing \$2.98. How much money will he need for both?

17. Ralph has \$3.45 and wants to buy a "Home Toymaker" costing \$1.79. If he buys it, he will have how much money left?

18. Donald can buy an "Erector" that will make 152

models for \$2.17, or one that will make 113 models for \$1.29. How much can he save by buying the cheaper set? How many more models will the larger set make?

19. James wanted a "Meccano" set costing \$2.69. He has \$1.85 in his bank. How much more money will he need?

20. Mr. Barnes bought one of his boys a "Meccano" set costing \$4.39 and the other an "Erector" set costing \$5.28. How much did both cost?

SOURCES OF PROBLEM MATERIAL

Toy-makers and any of the large mail-order houses will be glad to send catalogs of their toys. From these catalogs large charts can be made showing the pictures and actual prices, and problems of actual interest can easily be made. Thus you can make charts of a "Toy Shop"; "Toys Every Boy Wants"; "Toys every Girl Wants"; "Erector Sets"; "Meccano Sets"; "The Humpty Dumpty Circus"; "Gilbert Wheel Toys"; "Conver's Toys from Toy-Town"; "Balls, Tops, Marbles, and Kites"; "Mechanical Toys that Go"; "Dolls and Dolly's Supplies"; "Musical Toys"; and so on. All such work is not only much more interesting and instructive than many of the book problems about adult activities, but they lead the child to use his knowledge of arithmetic in his own personal affairs.

While the recent textbooks are filled with many good problems brought down to the child's needs

and interests, to supplement them with problems suggested by the season at which the topics are taught adds greatly to the interest in the subject, especially in the lower grades.

PROBLEMS GROUPED ABOUT SOCIAL ISSUES

The various seasons and the activities of the children often suggest interesting groups of problems that not only furnish a motive for arithmetic, but lead to the habit of using it in their actual activities. The following groups based upon "A Hallowe'en Party," "A School Picnic," and "Christmas Shopping," illustrate a few such possibilities.

A Hallowe'en Party

1. The second and third grade children had a Hallowe'en party. There were 38 children from the second grade and 36 from the third grade. How many children in the party?
2. Frank brought 26 apples, Harry brought 28, and Donald brought 35. How many apples had they?
3. If each child had an apple, how many of the 89 apples were left?
4. The third grade furnished cider for the party costing \$2.25 and the second grade class brought 12 pumpkins that cost \$1.80. How much more did the cider cost than the pumpkins?
5. How much did both pumpkins and cider cost?
6. A committee of boys and a committee of girls from each class collected doughnuts for the party. The

girls in the second grade brought 83 and the boys 69. In the third grade the girls brought 78 and the boys 74. How many doughnuts did they have for the party?

7. The boys and girls collected money to pay for the refreshments and decorations which they had to buy. They collected \$5.21 and spent \$4.38. How much was left?

8. They used the 83¢ left, with 65¢ which they got for the doughnuts and cider that were not used, to send some flowers to a sick classmate that could not be at the party. How much did they have for flowers?

A School Picnic

1. The second and third grade classes had a picnic in the woods one Saturday. There were 35 pupils from the second grade and 37 from the third grade. With their 2 teachers, how many were in the party?

2. There were 38 boys in the party. How many girls were there in the party?

3. One teacher collected the total amount of the carfare, which was \$14.80. The other collected the money for the refreshments, which was \$18.50. How much was the total expense of the trip?

4. How much more did the refreshments cost than the carfare?

5. The children found hickory nuts and walnuts on the trip. They gathered 38 quarts of hickory nuts and 27 quarts of walnuts. How many quarts of both did they gather?

Christmas Shopping

1. Mary and her mother went shopping to buy Christmas presents. At one store they bought a doll for

little sister and a kiddie horse for little brother. The doll cost \$2.38 and the kiddie horse cost \$1.79. How much did both cost?

2. How much more did the doll cost than the kiddie horse?

3. Mary bought her father a tie costing \$1.25 and a handkerchief costing 36¢. How much did both cost.

4. Mary's mother bought a Christmas tree for 75¢ and paid \$1.48 for trimming. What was the cost of the tree and trimming?

5. Mary spent 85¢ and her mother spent \$1.56 for presents to send to some cousins. How much did both spend?

6. Father was shopping, too, that day and bought Mary a doll costing \$2.89 and a set of doll furniture costing \$2.47. How much did both cost?

7. He looked at two construction toy sets from which outdoor wheel toys can be made. One cost \$8.38 and the other cost \$6.95. How much more did the more expensive one cost?

8. For mother he bought a dresser set costing \$3.18 and a serving tray costing \$1.57. How much did both cost?

THE MULTIPLICATION TABLES

Multiplication is to save adding when the addends are all equal. Hence the tables should be so developed as to bring out this fact. Most textbooks of the past, and many at present, develop the tables by counting by 2's, 3's, 4's, and so on, but this does not so fully bring out the meaning and use of multi-

plication as the development by addition. The counting method develops the table of 2's, 3's, 4's, etc., while the adding method develops the "2-times," "3-times," "4-times" tables.

THE "2-TIMES" TABLE

These are but the "doubles" of the forty-five primary facts of addition and hence are already known. They are

1	2	3	4	5	6	7	8	9
$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{4}{8}$	$\frac{5}{10}$	$\frac{6}{12}$	$\frac{7}{14}$	$\frac{8}{16}$	$\frac{9}{18}$

Instead of saying 1 and 1 are 2; 2 and 2 are 4; etc., the child should see that he can as well say "two 1's are 2," "two 2's are 4," "two 3's are 6," and so on.

THE CORRESPONDING DIVISION FACTS

There are two meanings and uses of division. Thus from $2 \times 5 = 10$, we have "half of 10 is 5," or "there are two 5's in 10." If division follows each table of multiplication, it is only the first type that is useful until several tables have been studied; for, now, the answer to each question of the second type would be "2."

It is not only useful, but it gives a variety to the drill upon multiplication to ask, "Half of 10 is what?" "Half of 12 is what?" and so on.

The only teaching and explanation needed is that one of the two equal parts of a number is half of it,

just as one of the two equal parts of an apple or of a stick of candy is one half of it.

To answer the question " $\frac{1}{2}$ of 12 = what?" the pupil simply recalls which of the groups of two equal numbers made 12.

Drill upon:

$$\begin{array}{lll} \frac{1}{2} \text{ of } 8 = ? & \frac{1}{2} \text{ of } 6 = ? & \frac{1}{2} \text{ of } 18 = ? \\ \frac{1}{2} \text{ of } 14 = ? & \frac{1}{2} \text{ of } 16 = ? & \frac{1}{2} \text{ of } 4 = ? \end{array}$$

THE "3-TIMES" TABLE

Have the pupils write down and find by addition the following sums:

1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{6}{3}$	$\frac{7}{3}$	$\frac{8}{3}$	$\frac{9}{3}$
3	6	9	12	15	18	21	24	27

Show that by remembering these sums much time is saved when wanting a sum of three equal numbers. Ask the cost of three articles if each costs 1¢, 2¢, 3¢, and so on to 9¢, and let the pupil answer from seeing the above sums.

Show a "new way" to write the above sums. Thus, asking, "How many 1's?" write it $3 \times 1 = 3$; "How many 2's?" write it $3 \times 2 = 6$, and so on. Omit the use of the word "times" until the pupil pictures the expression $3 \times 8 = 24$ as three 8's added as above.

Show the advantage of knowing the "3-times"

table by having a problem solved in two ways as follows:

Harry earned \$1.75 each week for three weeks. How much did he earn in all?

Finding out by adding:

$$\begin{array}{r} \$1.75 \\ 1.75 \\ 1.75 \\ \hline \$5.25 \end{array}$$

Finding out by a new way:

We write \$1.75 but once. The 3 tells how many we are to "think." Think three 5's are 15. Write 5, carry 1. Think three 7's are 21, and 1 are 22. Write 2, carry 2. Think three 1's are 3 and 2 are 5. Write 5.

Solve problems in both ways in order to show both the meaning of multiplication and its advantage in saving time.

Write the tables

$$\begin{array}{lll} 3 \times 1 = 3 & 3 \times 4 = 12 & 3 \times 7 = 21 \\ 3 \times 2 = 6 & 3 \times 5 = 15 & 3 \times 8 = 24 \\ 3 \times 3 = 9 & 3 \times 6 = 18 & 3 \times 9 = 27 \end{array}$$

The multiplier is written first and shows how many times the multiplicand was added to get the product. It is better to omit the word times at first and read the above expression as "three 1's are 3."

For convenience, however, in written work when the multiplicand is large the word "times" must be

developed and used. So from the development, show that the 3 shows that the 1's, 2's, 3's, etc., were written 3 times to add, and that we may say "3 times 1" instead of "three 1's."

CORRESPONDING DIVISION FACTS

In drilling upon the multiplication tables, you may ask such questions as "What is one of the three equal numbers in ——?" using 6, 9, 12, 15, and so on to 27. Then tell the children that a shorter way to say "one of the three equal numbers in" is to say "one-third of"; and that a shorter way to write it is " $\frac{1}{3}$ of," thus $\frac{1}{3}$ of 6 = 2, $\frac{1}{3}$ of 9 = 3, etc. Drill upon:

$\frac{1}{3}$ of 12	$\frac{1}{3}$ of 18	$\frac{1}{3}$ of 15
$\frac{1}{3}$ of 21	$\frac{1}{3}$ of 24	$\frac{1}{3}$ of 27

The drill may also include such problems as, "If 3 oranges cost 18¢, how much is that for each?" The pupil's thought about it should be that "one of the three" will cost $\frac{1}{3}$ of 18¢, or 6¢.

CHARTS FOR ORAL DRILL

From colored pictures taken from catalogs and magazines, charts may be made containing articles costing from 1¢ to 9¢ inclusive that may be used for all the multiplication tables. This is done not only for drill purposes but to bring out the meaning and use of multiplication. Thus, "The Bakery" may have

Bread	9¢	Cream puffs	6¢	Doughnuts	3¢
Small pies	8¢	Ginger bread	5¢	Large cookies	2¢
Jelly rolls	7¢	Hard rolls	4¢	Small cookies	1¢

Ask such questions as, "How much will 3 loaves of bread cost?" "How much will 3 cream puffs cost?"

A "Candy Shop" may have

Stick candy	1¢	Molasses taffy	5¢
Caramels	2¢	Almond bars	6¢
Lollipops	3¢	Milk chocolate	7¢
Butter scotch	4¢	Peanut brittle	8¢

Large Milk Chocolate Bars 9¢

A "Fruit Store" may have

Grapefruit	9¢	Peaches	5¢
Large oranges	8¢	Bananas	4¢
Large pears	6¢	Small apples	3¢
Large apples	7¢	Plums	2¢

MISSING-NUMBER DRILLS

Just as the "missing-number" drills in addition gave valuable drill in addition and at the same time prepared for subtraction, so similar drills in multiplication are not only valuable in fixing multiplication but they prepare for division. Thus following the "3-times" table, give the following drill:

$3 \times \dots = 15$	$3 \times \dots = 18$
$3 \times \dots = 12$	$3 \times \dots = 6$
$3 \times \dots = 24$	$3 \times \dots = 27$
$3 \times \dots = 9$	$3 \times \dots = 21$

This is only another way of asking the fact under the form of $\frac{1}{3}$ of 15 = ? $\frac{1}{3}$ of 12 = ? $\frac{1}{3}$ of 24 = ? and so on. But it is valuable for the child to know the meaning of the symbols and the facts when written in various forms.

INTERCHANGING MULTIPLIER AND MULTIPLICAND

Show objectively that $2 \times 3 = 3 \times 2$, $3 \times 4 = 4 \times 3$, etc. Thus,

• • • in columns, there are three columns of 2 each;
 • • • that is, there are 3×2 . In rows, there are three in a row and 2 rows, or 2×3 . In each new table, then, part of the facts are known. Thus $2 \times 4 = 4 \times 2$, $3 \times 4 = 4 \times 3$, etc., so these two facts are known from the "2-times" and "3-times" tables.

Show too that $3 \times 2\text{¢}$ is just as much as $2 \times 3\text{¢}$. In the problems that the pupil meets, he will have occasion to find $15 \times 3\text{¢}$ as often as $3 \times 15\text{¢}$.

To find the cost of 3 balls at 15¢ each, the multiplier is 3, and the problem is one in which the multiplier is not only logically, but actually 3. While in the problem "Find the cost of 15 apples at 3¢," the multiplier is really 15 and the solution is $15 \times 3\text{¢}$, a two-figured multiplier. But to get the *number* of cents in the answer, 3×15 is found. That is, 3 becomes the *actual* multiplier. To avoid developing a wrong notion of the real meaning of multiplication, the solution should be written as in the margin:

$$\begin{array}{r} 15 \times 3\text{¢} = 45\text{¢} \\ 3 \\ \hline 45 \end{array}$$

That is, the pupil should see that the real solution is 15×3 , but to get the product, 3×15 gives the same as 15×3 .

A NEW HABIT NEEDED IN CARRYING

In addition the pupil has formed the habit of adding the number carried to the first addend. In multiplication he has to multiply before carrying. This presents two difficulties: (1) he has to form a new habit; and (2) he has to hold in mind the number to be carried while finding the product. This takes greater concentration than was needed in addition. The child should be shown *why* he must add after multiplying and then form the *habit* of doing it.

To show *why*, find a result both by addition and multiplication. Thus, find 3×275 .

275	
275	275
<u>275</u>	<u>3</u>
825	825

Although in adding, he carries 1 to 7 before adding the other two 7's, he sees that it is really added to the three 7's, hence in multiplication he must first find the three 7's before adding. To form the *habit*, there are two types of oral drill that are useful. (1) They are such as $3 \times 4 + 1$, $3 \times 6 + 2$, etc.; and (2) add 2 to 3×4 , add 1 to 3×2 , etc. In the first he does not have to hold in mind a number while finding a

product as he does in actual written work. In the second the work is more nearly what he has to do in written work.

This form of drill should follow each table. After the "3-times" table but 1 or 2 will need to be added. So the following are sufficient.

$1+3\times 1$	$1+3\times 3$	$2+3\times 1$	$2+3\times 3$
$1+3\times 4$	$1+3\times 9$	$2+3\times 4$	$2+3\times 9$
$1+3\times 2$	$1+3\times 7$	$2+3\times 2$	$2+3\times 7$
$1+3\times 6$	$1+3\times 8$	$2+3\times 6$	$2+3\times 8$

In dictating to a class, use both forms of statement, viz.: "What is 3×4 and 1?" "What is 1 added to 3×4 ?" or "What is 3×4 plus 1?" "What is 1 plus 3×4 ?"

Some Practical Uses of the "2-Times" and "3-Times" Tables

1. Mr. Brown bought John and his brother each a \$4.98 hand car. Find without adding, how much both cost.
2. Walter and his sister each got a coasting sled for Christmas. They cost \$3.67 each. Without adding, find what both cost.
3. Mr. Barnes bought 3 boxes of apples. Each box was marked "96 to the box." Find without adding how many apples in the three boxes.
4. Mary picked up 3 baskets of apples which her father let her sell. She sold them for \$1.35 a basket. How much did she get for all?

5. Frank had 3 hens for which he was offered \$1.85 apiece. At that price, how much were the three worth?

6. Mary's mother bought 3 yards of gingham to make her a dress. It cost \$0.68 a yard. How much did the three yards cost?

7. Harry raised 3 baskets of carrots, which he sold for \$0.75 a basket. How much did he get for them?

8. Donald gathered 3 baskets of walnuts, which he sold for \$1.65 a basket. How much did he get for them?

The "4-Times" and "5-Times" Tables

Since the development of the tables is to fix the meaning of the tables, it is not necessary that all should be found through addition, but enough should be found to make clear the meaning and use of the facts. It is well, then, to develop the 4-times and 5-times tables by addition. Thus,

1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
$\frac{1}{4}$	$\frac{2}{8}$	$\frac{3}{12}$	$\frac{4}{16}$	$\frac{5}{20}$	$\frac{6}{24}$	$\frac{7}{28}$	$\frac{8}{32}$	$\frac{9}{36}$

$4 \times 1 = 4$	$4 \times 4 = 16$	$4 \times 7 = 28$
$4 \times 2 = 8$	$4 \times 5 = 20$	$4 \times 8 = 32$
$4 \times 3 = 12$	$4 \times 6 = 24$	$4 \times 9 = 36$

NOTE.—Develop the "5-times" table in the same way.

TEACHING ONE-FOURTH AND ONE-FIFTH

Just as one of three equal numbers in a number is called "one-third of it," show the pupils that one of

the four equal numbers that make up a number is called "one-fourth of it," and that one of the five equal numbers, is "one-fifth of it." Then drill as suggested under the "3-times" table, asking, "What is one of the four equal numbers that make —?" Then give such drills as

$\frac{1}{4}$ of 8 = ? $\frac{1}{4}$ of 24 = ? $\frac{1}{5}$ of 5 = ? $\frac{1}{5}$ of 45 = ?
 $\frac{1}{4}$ of 20 = ? $\frac{1}{4}$ of 32 = ? $\frac{1}{5}$ of 30 = ? $\frac{1}{5}$ of 25 = ?
 $\frac{1}{4}$ of 36 = ? $\frac{1}{4}$ of 28 = ? $\frac{1}{5}$ of 20 = ? $\frac{1}{5}$ of 10 = ?
 $\frac{1}{4}$ of 16 = ? $\frac{1}{4}$ of 4 = ? $\frac{1}{5}$ of 15 = ? $\frac{1}{5}$ of 35 = ?

Also use the "missing-number" drill suggested under the "3-times" table.

Some Uses of the "4-Times" and "5-Times" Tables

1. Frank's mother sent him to the store for a 4-pound roast. At 38¢ per pound, how much money should she give him?

2. A class wants 5 dozen apples for a party. At 45¢ per dozen, how much will they cost?

3. To trim a Christmas tree a class wanted 5 boxes of tinsel. At 27¢ per box, how much will it cost?

4. John saved 45¢ each week for 5 weeks to get money to buy a coaster. How much did he save?

5. Frank and his sister picked up 5 baskets of wind-fall apples which they sold for 85¢ a basket. How much money did they get for them?

6. Ralph kept hens and sold eggs. One week he sold 5 dozen at 68¢ a dozen. How much did he get for all?

7. Walter sold a 4-pound chicken at 38¢ a pound. How much did he get for it?

8. Donald earned \$1.85 a week delivering papers. How much can he earn in 5 weeks?

9. Mary went Christmas shopping with her mother. They bought 4 dolls to send to some cousins. They cost \$1.78 each. How much did all cost?

10. To trim a Christmas tree, Helen wants 15 red tissue paper bells costing 4¢ each. How much will they cost?

NOTE.—Note that the solution is $15 \times 4¢$, but that the actual multiplication is 4×15 . The rest of this list are of the same general type.

11. To trim a tree for the school, the class wanted 18 large red tissue paper bells that cost 5¢ each. How much will all cost?

12. Helen had an Easter party. She wanted a pretty Easter card for each child at the party. If they cost 5¢ each, how much will cards for 17 children cost?

13. Lucile pays 5¢ to ride to school each day and the same to ride back. One month she rode 41 times, counting both going and coming. How much did it cost her?

14. Lucy was going to have a party and her mother sent her to the bakery for 36 little cakes costing 4¢ each. How much money should she give her?

15. Some boys were going on a hike. They had 24 sandwiches made for them to take for lunch. At 5¢ each, how much did they cost?

16. One week each pupil in a class of 39 gave 5¢ to the Junior Red Cross. How much did the whole class give?

17. Mary's mother gave her 5¢ each day for helping her dry the dinner dishes. There are 31 days in December. How much did she earn that month?

18. John got 4¢ each day for feeding the hens and bringing in the eggs. How much did he earn in December?

THE DIVISION FACTS

As was shown on a preceding page, the primary facts of division come directly from the multiplication facts. If the pupil knows a multiplication fact and its meaning, he must know the division fact if it is properly asked. Thus from two 7's are 14, and seven 2's are 14, he will tell you the answer to "How many 7's in 14?" and "How many 2's in 14?" But if the question is asked in new language, as "14 divided by 2 equals what?" he cannot answer, for the question calls up no fact familiar to him. With each table, he has learned one meaning of division. Now that he knows four tables, he is ready for such questions as "How many 2's in 4?" "How many 3's in 6?" "How many 4's in 24?" etc. While he must understand the meaning and answer to each type of division question, the type of question shown here seems the simplest and most common as a foundation for the written process.

THE NOTATION OF DIVISION FACTS

The child has found $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ of numbers within the tables, but this has not even been called division. The only notation he has had is $\frac{1}{3}$ of $6 = 2$.

He must now become familiar with both $6 \div 3 = 2$ and $\begin{array}{r} 2 \overline{)6} \\ 3 \end{array}$. The second form should be the one used

for drill purposes, for it is the form he will see in written work. As a fact is asked, show him how to write it. Thus, first ask, "How many 2's in 6?" and

when he answers, write it on the blackboard
$$\begin{array}{r} 2 \overline{) 6} \\ 3 \end{array}$$

saying, "This simply says that there are three 2's in 6," pointing to the figures as you say this. While this question gives but one of the two meanings of division—the *measuring* idea—it seems to be the one to ask in connection with written work, and it is the one that comes naturally from the meaning of multiplication.

THE PLACE OF THE QUOTIENT IN SHORT DIVISION

In many schools and in many of the recent textbooks, the quotient in short division is placed above the dividend as in long division. The only argument for such a form is that the pupil forms the *habit* in short division so that it makes it easier for him to do so in long division where such a position is important. But there are much stronger arguments for placing it below than the single argument for placing it above. In the first place, sooner or later he *must* form the habit of placing the quotient below; so the time to form the habit of placing it above is lost, granting that it really took any extra time to form the two habits. In the second place, actual practice does not show that pupils trained to write the

quotient below in short division experience any awkwardness in placing it above in long division.

The reasons for writing the quotient below in short division are many. The following illustrations should convince any teacher that in the actual use of short division, since it usually occurs with other processes, the quotient *must* be written below.

EXAMPLE 1. Find the average of 15, 24, 20,

15 and 21.

24 Here the space above has been used, so the
20 quotient must be written below or time
21 wasted to rewrite the sum before dividing.

$$\begin{array}{r} 4 \overline{)80} \\ 20 \end{array}$$

EXAMPLE 2. John was offered a bicycle costing \$26.80 for $\frac{1}{4}$ less than cost. For how much could he have bought it?

Here the quotient is needed below in

4) \$26.80 order to subtract without rewriting it.
6.70
\$20.10

EXAMPLE 3. How much will $\frac{5}{8}$ of a yard of lace cost at \$5.84 per yard?

8) \$5.84	or	\$5.84
.73		5
5		8) \$29.20
\$3.65		\$3.65

In either case the quotient had to be placed below. It should be clear, then, to teachers that the

quotient could not be placed above in the everyday and business uses of short division. For that reason, such a form should not be taught in school. It is because so many forms are taught in schools that cannot be used in life that the arithmetic teaching of our schools is so severely criticized by the business man as "impractical" and "useless."

THE TWO MEANINGS OF DIVISION ILLUSTRATED

Division means either a measuring of the dividend by the divisor to see how many times the dividend contains the divisor, or a dividing it into parts, depending upon the problem. The first is called *measurement*, and the second *partition*. They are illustrated by the following examples:

EXAMPLE 1. How many oranges at 3¢
 $3\cancel{\text{¢}}\overline{)12\cancel{\text{¢}}}$ each can Frank buy for 12¢?

4 Here we are measuring 12¢ by 3¢. The quotient 4 shows that 12¢ is 4 times as large as 3¢. That is, it would take $4 \times 3\text{¢}$ to make 12¢; or, that from 12¢, 3¢ could be taken 4 times.

The quotient 4, then, is not oranges, but a pure abstract number showing the relation of the dividend to the divisor. But since each 3¢ will buy an orange, 12¢, which is 4 times as large, will buy 4 oranges.

The proper language here is "will contain." That is, 12¢ will contain 3¢, 4 times. The pupil must see that the quotient 4 simply shows that 12¢ is 4 times as much as 3¢.

EXAMPLE 2. Frank bought 3 oranges for 12¢. How much was that for each?

Here each orange would cost $\frac{1}{3}$ as much
 $3 \overline{)12\text{¢}}$ as 3. That is, the cost 12¢ is divided into 3
4¢ equal parts and each part is 4¢.

Here the language to use is "divided by" or "one-third of." In short division, the latter expression should be used. When the divisor is large, as 125, it becomes awkward to use this language; that is, it is awkward to say "one one-hundred-twenty-fifth of," and for that reason the first form is used. The advantage of the second form in short division is that it really describes what is actually done. In this kind of division, "will contain" can never be used. In many schools, the expression "goes into," often pronounced "gus-sin-tu," is used for both kinds of division. Such expressions, of course, have no meaning and should not be used. In both $\$12 \div \3 and $\$12 \div 3$ the sign may be read "divided by," but such expressions as " $\$3$ divided into $\$12$ " and " 3 divided into $\$12$ " should not be used.

WRITTEN WORK IN SHORT DIVISION

The development of the written processes is not so important as the careful grading of the exercises in order that the pupil may gradually "grow" into the habits of performing them.

The first problems of short division should be those

in which each digit of the dividend will contain the divisor.

If the partition idea of division has been developed and used, then the process may be rationalized by such problems as the following:

EXAMPLE 1. John paid 48¢ for 2 balls. How much was that for each?

This is the way we find out:

48¢ is equal to 4 dimes and 8¢. Half of 4 dimes is 2 dimes. Write 2. Half of 8¢ is 4¢.

$$\begin{array}{r} 2 \overline{)48\text{¢}} \\ 24\text{¢} \end{array}$$
 Write 4¢.

EXAMPLE 2. John paid 56¢ for 2 balls. How much was that for each?

This is the way we find out:

56¢ is equal to 5 dimes and 6¢. Half of 5 dimes is 2 dimes and 1 dime not divided.

$$\begin{array}{r} 2 \overline{)56\text{¢}} \\ 28\text{¢} \end{array}$$
 The dime not yet divided and 6¢ make 16¢.
 Half of 16¢ is 8¢.

These two problems show the rationalization with money as a dividend. To rationalize it with a dividend of *any* kind, the decimal place-value feature of our notation must be understood. Thus, to divide 56 by 2, we would proceed as follows:

56 is equal to 5 tens and 6 ones. Half of 5 tens is 2 tens and 1 ten (10) undivided. 1 ten and 6 ones are 16 ones. Half of 16 ones is 8 ones.

$$\begin{array}{r} 2 \overline{)56} \\ 28 \end{array}$$

It is a question as to whether it is worth while to rationalize the process any further than by the use of money. When the child can divide such exercises as those given above, he may simply use the process with any kind of numbers through habit instead of through rationalization.

DEVELOPING THE PROCESS THROUGH MEASUREMENT

Even less rationalization than that shown above may be used. This requires a different type of question, and is based upon the measuring idea of division.

EXAMPLE 1. John has 48¢. How many marbles can he buy at 2¢ each?

This is the way we find out:

We must find how many 2¢ pieces are in 48¢;
$$\begin{array}{r} 2 \overline{)48} \\ 24 \end{array}$$
 that is, how many 2's in 48. Beginning at the left, think how many 2's in 4. Write 2. Then think how many 2's in 8. Write 4.

As the first question is asked, cover the 8 so that the pupil sees only $2 \overline{)4}$.

This, it will be noticed, is no rationalization at all, but a mere presentation of the "how," for the real measurement questions are, "How many 2's in 40?" "How many 2's in 8?" which are too difficult for the child to see and answer.

EXAMPLE 2. John has 56¢. How many marbles at 2¢ each can he buy?

This is the way we find out:

Think how many 2's in 5. There are 2 and 1 remaining not divided. Write 2. 1 of the 5's and 6 in the place at the right of it make 16 not yet divided. How many 2's in 16? Write 8. As the first question is asked, cover the 6 so the child sees but $2 \overline{)5}$. Then, showing the 6, have him picture the 1 and 6 to the right of it, making 16.

This is a mere presentation of the "how" and no rationalization of the "why." But since the meaning of division is brought out through the problems, and not through the "why" of the steps in the process itself, the second method (the measurement method) is just as pedagogical as the first, and is the one generally preferred.

Graded Drills

$2 \overline{)46}$	$3 \overline{)63}$	$4 \overline{)48}$	$5 \overline{)55}$
$2 \overline{)84}$	$3 \overline{)96}$	$4 \overline{)84}$	$5 \overline{)155}$
$2 \overline{)28}$	$3 \overline{)39}$	$4 \overline{)168}$	$5 \overline{)200}$
$2 \overline{)104}$	$3 \overline{)126}$	$4 \overline{)124}$	$5 \overline{)305}$
$2 \overline{)128}$	$3 \overline{)189}$	$4 \overline{)208}$	$5 \overline{)400}$
$2 \overline{)146}$	$3 \overline{)153}$	$4 \overline{)288}$	$5 \overline{)255}$

2) <u>162</u>	3) <u>219</u>	4) <u>244</u>	5) <u>455</u>
2) <u>184</u>	3) <u>246</u>	4) <u>328</u>	5) <u>405</u>
2) <u>36</u>	3) <u>45</u>	4) <u>56</u>	5) <u>65</u>
2) <u>52</u>	3) <u>54</u>	4) <u>64</u>	5) <u>70</u>
2) <u>78</u>	3) <u>72</u>	4) <u>72</u>	5) <u>85</u>
2) <u>94</u>	3) <u>84</u>	4) <u>92</u>	5) <u>95</u>
2) <u>116</u>	3) <u>105</u>	4) <u>104</u>	5) <u>115</u>
2) <u>132</u>	3) <u>117</u>	4) <u>112</u>	5) <u>120</u>
2) <u>156</u>	3) <u>135</u>	4) <u>136</u>	5) <u>145</u>
2) <u>174</u>	3) <u>147</u>	4) <u>140</u>	5) <u>165</u>

ORAL DRILL CHART

As seen in the exercises above, the pupil must know not only the "division tables," but he must know "how many, and how many remaining" for each divisor. Thus in dividing by 5, he will meet "how many 5's" in any number from 5 to 49 inclusive. And for any other divisor a similar question for all numbers from the divisor itself to 10 times the divisor will be raised. So drill in the mere division

tables is not sufficient for efficient work in written division. A chart, a specimen of which is given below, should be arranged for each divisor from 2 to 9.

In making the following chart for division by 5, it is seen that 45 numbers are needed. Since the factors of 45 are 5 and 9, arrange a chart divided into 9 divisions by 5 divisions, and beginning with 5, place each number from 5 to 49 in a space, arranging them miscellaneously.

Drill Chart for Division by 5

5	27	39	19	43	31	16	47	23
10	37	6	40	30	22	34	17	32
42	11	29	21	7	45	25	48	18
36	41	12	35	44	14	8	24	49
28	20	38	13	26	33	46	15	9

USING THE CHART

In using the chart, the pupil will give the quotient and the remainder. Thus when the teacher points to 17, he will say "3 and 2 remaining." To save time, he may say "left" instead of "remaining." The table brings up every possible fact that can ever be met when dividing by 5 and hence insures a more thorough drill upon *all* the needed facts than is likely to result from the written exercises that are made up at random.

By making the drill into a "game," pupils enjoy it. Thus let them play O-U-T Spells Out, the game being

to see who can give the whole 45 quotients and remainders without being spelled out, a letter being called in order as errors are made. It is advisable to spend a few minutes daily upon this type of drill for each of the tables.

PROBLEM CHARTS

The "Bakery," "Fruit Store," and "Candy Shop" charts suggested for drill upon the multiplication tables may be used for division problems as well. Thus, how many doughnuts (they are 3¢ each on the chart) can Helen buy for 9¢, for 15¢, for 18¢, etc. Or, how many can she buy for 10¢? Here the pupil will say, "Three and 1¢ not spent." Such problems with small numbers bring out clearly the meaning and use of division.

Multiplication and Division Problems

Problems should be given in which some involve multiplication and some involve division in order that the pupil may judge from the problem which process must be used.

1. Five boys went on a picnic one Saturday. It cost them \$1.34 each. How much did it cost for the 5 boys?
2. Five boys went on a picnic one Saturday. The entire cost of the trip was \$6.35. That was an average of how much for each boy?
3. Helen's mother paid \$2.45 for a 5-pound duck. How much was that a pound?

4. Helen's mother bought a 5-pound roast at 42¢ a pound. How much did it cost her?

5. Four boys living by a nice hill for coasting decided to buy a large coaster for \$9.60 and share the expense equally. How much will each have to pay?

6. A third grade class are going to have a Hallowe'en party. For this party they want 3 gallons of cider. Cider costs 65¢ per gallon. How much will the cider cost?

7. Mary's older sister found the cost of their Thanksgiving dinner to be \$6.45. There were 5 in the family. Mary found the average cost for each member of the family. How much was it?

8. Frank wants to earn enough money in 4 weeks to buy a coaster costing \$5.36. How much will he have to average each week?

9. Ralph earns \$1.65 per week delivering papers. How much can he earn in 4 weeks?

10. Walter had 4 rabbits which he sold for \$5.80. How much was that for each?

THE REMAINING MULTIPLICATION FACTS

But ten new facts remain. The easiest group of the remaining facts is the "9-times" table. Knowing 2×9 , 3×9 , 4×9 , and 5×9 from the tables already learned, the pupil knows 9×2 , 9×3 , 9×4 , and 9×5 . Write them as,

$9 \times 2 = 18$ Call attention to the fact that the
 $9 \times 3 = 27$ left-hand digit of each product is 1 less
 $9 \times 4 = 36$ than the number taken 9 times and that
 $9 \times 5 = 45$ the sum of the two digits of the product

is 9. Then tell him that this is so of 9×6 , 9×7 , 9×8 , and 9×9 , and he can write and remember the "9-times" table at once.

The remaining facts need more drill. They are:

$$6 \times 6$$

$$6 \times 7$$

$$6 \times 8$$

$$7 \times 7$$

$$7 \times 8$$

$$8 \times 8$$

The corresponding division facts are developed and drilled upon just as those already discussed. Use all games and devices suggested for former drills and make up problems from charts and from the child's own environment as suggested for the first part of the tables. The partition question of division may be used after each table as suggested in the first five tables. Here the questions are $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, and $\frac{1}{9}$ of all products within the tables. Also use the "missing-number" drills.

PROBLEMS IN MULTIPLICATION

The pupil knows from the meaning of multiplication that 3 tool chests at \$2.75 will cost $3 \times \$2.75$, and also that the answer can be found by writing \$2.75 three times and adding. Likewise he knows that the cost of 48 marbles at 2¢ is $48 \times 2¢$, and that it can be found by writing 2¢ forty eight times and adding it. In such problems he should see that $2 \times 48¢$ gives the same product as $48 \times 2¢$.

In order that he does not lose the meaning of

multiplication and think that he can multiply by 2¢, he should use abstract numbers in finding the product. Such problems should be stated as follows:

$$\begin{array}{r} 48 \times 2¢ = 96¢ \\ \underline{2} \\ 96 \end{array}$$

In analyzing this, he knows that the solution is $48 \times 2¢$, just as if it had been, "What is the cost of 3 marbles at 2¢ each?" in which case it would be $3 \times 2¢$. But show him that the *number* of cents is the product of 48×2 , which is the same as 2×48 just as $4 \times 2 = 2 \times 4$, so the smaller number is taken as the actual multiplier. In short, when the analysis of a problem leads to the discovery that the process is multiplication, the pupil should use abstract numbers in getting the product and select the smaller factor as the multiplier.

Type Problems

1. In a third grade of 38 pupils, each pupil gave 8¢ to the Junior Red Cross. How much did the whole class give?

Solution. $38 \times 8¢ = 304¢ = \$3.04$

$$\begin{array}{r} \times 8 \\ \hline 304 \end{array}$$

2. Some children are going on a picnic. There are to be 26 in all. If they allow 3 sandwiches for each, how many must they take?

3. A third grade of 34 pupils had a party. Each brought 7¢ to pay for the refreshments. How much did they all bring?

4. If each pupil of a class of 38 uses 6 pencils during the year, how many will be used by all?

5. At a party each child had 5 pieces of fudge. How many pieces had to be made for a party of 18?

6. If each member of a class of 35 gives 8¢ toward buying and trimming a Christmas tree, how much money will they have for it?

7. To trim a Christmas tree the class wants 24 ornaments costing 2¢ each. How much will all cost?

8. They want 18 larger ones costing 5¢ each. Find what they will cost.

9. At a Hallowe'en party the class wants 45 doughnuts costing 3¢ each. Find the cost of all.

10. At the Hallowe'en party each of the 17 boys brought 5 apples. How many apples did they have?

MEASUREMENT PROBLEMS IN DIVISION

Through simple oral and written problems, the pupil should see clearly the two uses of division. In the measuring problems, have the pupils see that the quotient is a pure abstract number, without a name, showing how many times the divisor is contained in the dividend, or can be taken out of it. Thus, "How many 2¢ sticks of candy can Dorothy buy for 8¢?" The answer to the question is, "4 sticks"; but 4, and not "4 sticks," is the quotient. The 4 shows that 8¢ is 4×2 ¢. But since 2¢ will buy 1

stick and there are 4 of them in 8¢, 8¢ will buy 4 sticks. In the written work, do not allow the quotient labeled in such problems. Thus, in the problem "At 4¢ each, how many oranges can be bought for 96¢?" have it written as shown here:

$$\begin{array}{r} 4\cancel{\text{¢}}\overline{)96\cancel{\text{¢}}} \\ 24 \end{array}$$

$$\begin{array}{r} 4\cancel{\text{¢}}\overline{)96\cancel{\text{¢}}} \\ 24 \end{array}$$

24, the number of oranges. Not, 24 oranges. Have the pupils see that the 24 shows that 96¢ is 24 times as large as 4¢, and that since each 4¢ will buy an orange, 96¢ will buy 24 of them.

Type Problems

1. There are 36 pupils in a certain room. If they march 3 abreast, how many ranks will there be?
2. Helen has 56¢ to spend for Christmas cards. If she buys cards that are 4¢ each, how many can she buy?
3. A third grade class is preparing for a party. At 3¢ each, how many little cakes can they buy for \$1.62 which they have collected?
4. A class collected \$1.96 to buy oranges at 4¢ each for a party. How many can they buy?
5. A third grade class collected \$1.08 to buy colored balls to decorate a Christmas tree. At 6¢ each, how many can they buy? If they select smaller balls costing 4¢ each, how many can they buy?
6. At 7¢ each, how many red tissue paper bells can they buy for their Christmas tree, if they have 94¢ left for that purpose?

PARTITION PROBLEMS IN DIVISION

The pupil has fully as great a need of the partition problem in division as of the measuring problem. That is, he may need to find how many each will have when a number of things is divided equally among several, as he will to find how many times a given number contains a like number.

He has learned the meaning of $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and so on to $\frac{1}{6}$ of a group of objects within the tables. Thus, if 12 apples are divided equally among 3 children, each gets $\frac{1}{3}$ of them; if divided equally among 4 children, each gets $\frac{1}{4}$ of them; if among 6, each gets $\frac{1}{6}$ of them. Likewise, he should see that to find $\frac{1}{3}$ of a number of things, we divide by 3; to find $\frac{1}{4}$ we divide by 4; to find $\frac{1}{5}$, we divide by 5; and so on. This should now be extended to written work. And while he should recognize the meaning of the process, he should see that the answer is found in the same way in both kinds of division.

Type Problems

1. Three boys went on a nutting trip. They found 234 nuts in all, which they divided equally. How many did each get?

We are to find $\frac{1}{3}$ of 234. But if the development of division has been "How many 3's in 23?" use that form of thought here instead of " $\frac{1}{3}$ of 23 is how many?" That is, the pupil should think of the process of finding " $\frac{1}{3}$

of" merely as division, and use the same thought in both measurement and partition.

2. Three boys bought a coaster for \$7.95, and agreed that each should pay an equal amount. How much must each pay?

3. Mary picked 96 pansies. If she keeps $\frac{1}{3}$ of them, how many will that be?

4. Ralph had 72 marbles. He told his brother that he could have $\frac{1}{4}$ of them if he could tell how many that would be. How many is $\frac{1}{4}$ of 72?

5. John wanted an "Erector" set costing \$5.76. His father told him that he would give him enough to finish buying it when he saved $\frac{1}{4}$ of the price himself. How much must John save?

6. Helen wanted a dress that had cost \$6.48. But her mother got it at a special sale and saved $\frac{1}{4}$ of the price. How much did she save?

7. Five boys went on a hike and shared the expenses equally. The expenses were \$3.45. How much did it cost each?

8. Mary went on an automobile trip with her parents. There were 5 in the party and their lunch cost them \$3.80. How much was that for each?

9. Henry offered to sell Frank a used bicycle that cost \$25.50, for $\frac{1}{3}$ of the cost. For how much could Frank buy it?

10. Helen had a party. Counting herself, there were 8 at the party. They had a large box of candy containing 128 pieces. If they shared the candy equally, how many pieces did each get?

11. In 4 weeks, Henry sold \$7.36 worth of eggs from his hens. How much did that average per week?

12. The feed for his hens for 4 weeks cost him \$3.12. How much was that per week?

13. Mrs. Brown bought a remnant of gingham for \$1.95 to make Helen a dress. There were 3 yards of it. How much did it cost per yard?

14. In a third grade there were 42 pupils. If they divide up into 3 equal groups for a game, how many will there be in each group?

15. A farmer offered the third grade class 8 pumpkins for their Hallowe'en party for \$1.20. How much was that for each?

16. Helen went on an automobile trip with her father and mother. In 8 hours they drove 184 miles. That was an average of how many miles per hour?

PROBLEM MATERIAL FROM CURRENT INTERESTS

The completion of the primary number facts and the four fundamental processes to include short multiplication and short division opens up a large variety of human interests to which the pupils may apply their arithmetic. Problems made from such interests are important, not only to furnish drill in the computation involved, but to develop power to choose the process to apply and to lead to the habit of using the facts and processes. The trouble with so much of our school work is that it is not made to function in the life out of school.

The following list will no doubt suggest some activities of interest to third grade pupils of any school.

1. A Hallowe'en party.
2. A Thanksgiving dinner.
3. A nutting party.
4. A picnic party.
5. A coasting party.
6. A sleigh ride.
7. The Christmas tree.
8. Christmas shopping.
9. Raising pets.
10. Keeping chickens.
11. Making a garden.
12. Shopping for mother.
13. Going fishing.
14. Vacation trips.
15. Mother's birthday.
16. A birthday party.
17. Earning money.
18. Spending money.
19. Recreation and amusements.
20. The public playground.
21. Going to the park.
22. An automobile trip.
23. Helping mother.
24. The Junior Red Cross.
25. A camping trip.
26. A school entertainment.
27. Making toys.

MAKE-BELIEVE ACTIVITIES: PLAYING STORE

Pupils enjoy playing at some adult activity. "Playing Store" offers a wide range of variations to meet any stage of development. A pupil may be chosen clerk and take orders for articles, merely naming the cost of the amount bought. Or, the buyer may pay his bills in toy money, or have them charged and bills made out and sent.

The clerk should lose his position whenever he makes an error and the first one to find the error may become clerk. This will encourage pupils to follow each computation made, thus each one will be getting drill.

Suppose that the game is "Playing Vegetable Market." One pupil may sit in front of the class and, from a large chart containing pictures of vegetables, be the "storekeeper" and take orders. Some pupil stands and says, "Send me 4 quarts of lima beans and a peck of potatoes." The pupils all see from the chart the price of the beans per quart and the price of the potatoes per peck. Suppose that the beans are 14¢ per quart and the potatoes 58¢ per peck. All compute the amount of the bill. When the clerk has made the computation, she will stand and say, "The 4 quarts of lima beans will cost you 56¢, and the potatoes 58¢, making a total of \$1.14." If she gives a wrong product or a wrong sum, the first to find it becomes clerk. To give practice in subtraction, the buyer may say, "Take the amount

from a \$2 bill," or "from a \$5 bill," or any amount sufficient to pay the bill and leave some change. Suppose the bill is paid with \$2. All compute the change. Then the clerk says, "Your change is 86¢."

Pictures can be found anywhere for attractive "playing-store" charts or empty containers may be obtained to make attractive counters.

Thus, charts can be made for

1. The grocery store.
2. The bakery.
3. The butcher shop.
4. A fruit store.
5. A vegetable market.
6. A toy store.
7. A dry goods store.
8. A furniture store.
9. Boys' clothing store.
10. Girls' clothing store.
11. A gift shop.
12. A candy shop.
13. A lunch room.
14. A delicatessen.

ORDERING FROM A CATALOG

The "playing store" can be varied by having large charts to represent pages from toy catalogs. The pupils will select articles they want and write out an order as follows:

56 Forest Ave., Jackson, Mich.

May 16, 1922.

Miller, Brown & Co.,
Chicago, Ill.

Dear Sirs:

Kindly send me, by express, the following:

1 Erector, No. 6	\$8.98
1 tool chest, No. 4	5.79
2 box kites, 48¢ each	.96

Money order enclosed,	\$15.73
-----------------------	---------

Trusting that you will fill the order promptly, I am

Yours truly,

John Barnes.

The orders may be made to represent orders sent in the "playing-store" games. Thus all may play that they are "storekeepers" and may order supplies for any of the stores represented upon the charts. This will bring in both multiplication and addition, while a personal order from a catalog as shown above does not offer much chance for multiplication or the use of a wide range of numbers.

TYPE PROBLEMS FROM THE TOPICS SUGGESTED

The problems, of course, must be made to meet the needs and interests of the children and based upon actual experiences when possible. If they are "made-up" problems instead of actual experiences, they should still be real in the sense that they are prob-

lems that would likely be raised, and that they should conform to local conditions and prices.

A Hallowe'en Party

1. A third grade is going to have a Hallowe'en party. They have decided to have 8 dozen (96) doughnuts costing 2¢ each. How much will the doughnuts all cost?

2. They are to have 48 apples costing 3¢ each. Find the cost of the apples.

3. They are to have 3 gallons of cider costing 45¢ a gallon. How much will the cider cost?

4. For decorations they bought 9 pumpkins costing 15¢ each. How much did the pumpkins cost?

5. They bought 8 candles at 3¢ each to light the pumpkins. How much did the candles cost?

6. All other decorations such as autumn leaves, corn on the stalk, and pictures were gathered by the pupils without any further cost. Find the total cost of what they had to buy.

7. The children began guessing how much each would have to contribute. Ralph asked, "How much will 5¢ from each of us be?" There were 38 pupils. Tell him.

8. The pupils found that 5¢ from each would make but \$1.90. The teacher said that 3 nickels from each would not be enough. Find how much that would be by finding $3 \times \$1.90$.

9. The teacher said that it was so nearly enough that she would furnish the rest. If the whole cost was \$6.15 (found in problem 6) and the pupils paid \$5.70 (found in problem 8), how much will the teacher have to pay?

A Thanksgiving Dinner

1. A small Thanksgiving turkey weighing but 9 pounds will cost how much at 48¢ per pound?
2. A 6-pound duck at 45¢ per pound will cost how much?
3. How much will a dozen oranges and 2 pounds of mixed nuts cost if the oranges are 48¢ per dozen and the nuts 38¢ per pound?
4. Mary's mother estimated the cost of the dinner to be as follows: Turkey, \$5.25; vegetables, 45¢; bread and pies, 65¢; fruit and nuts, \$1.10; other expenses, \$1.25. Find the whole cost of the dinner.
5. If the Thanksgiving dinner for a family of 6 cost \$7.44, how much did that average for each?
6. Mary's mother said that she did not want the dinner to average more than 85¢ each for her family of 6. How much would this allow her?
7. If she allows but \$5.10 for the dinner and buys two ducks costing \$3.28, how much has she left for the rest of the dinner?

A Nutting Party

1. Eight boys went on a nutting party. Their carfare was 16¢ each. How much was it for the 8?
2. For lunch each boy had 2 sandwiches costing 7¢ each. How much did they cost for each boy? For all 8?
3. The entire lunch for all cost \$2.72. How much did that average for each?
4. In all they got 3 bushels of nuts. There are 32 quarts in a bushel. How many quarts did they get?

5. If the 96 quarts are divided equally, how many quarts will each boy get?
6. Two of the boys got there late and took but 6 quarts each as their share. How many quarts did that leave for the other 6 boys?
7. If the 84 quarts were divided equally between the 6, how many would that be for each?

A Picnic Party

1. Nine boys and girls went on a picnic. The cost of the lunch was \$3.42. How much was that for each?
2. They got a bus to take them out to the woods and return for them for \$2.25. How much was that for each?
3. Find what the bus fare (\$2.25) and the lunch (\$3.42) both cost?
4. If both cost \$5.67, find how much that was for each of the 9.
5. Compare your answer in 4 with the cost of each lunch found in problem 1 and each bus fare found in problem 2. Did you expect the answer in problem 4 to be the sum of the other two answers?

A Coasting Party

1. Some boys and girls had a coasting party one afternoon near Frank's home. Frank's mother had some hot chocolate and sandwiches for them. If she allows 2 cups of cocoa and 3 small sandwiches for each, how much of each must she make if there are 16 in the party?
2. Frank's coaster cost \$8.64 and Walter's cost \$7.98. How much more did Frank's cost?

3. Four boys owned a \$9.48 coaster together. How much was each share worth?

4. There were 4 coasters in all. One was the \$9.48 one, one cost \$8.64, one cost \$7.98, and a small one cost \$4.75. Find the cost of all.

A group of problems based upon some social activity of the season is not only more interesting, but such work is more likely to lead the pupil to use his knowledge of numbers in answering questions about the quantitative side of life than will the isolated problems that are less vital.

CHAPTER V

THE WORK OF THE FOURTH YEAR

The four fundamental operations with whole numbers should be stressed and fixed during the year. Pupils who do not know thoroughly these processes and all the fundamental facts involved, and do not have considerable skill in using them by the end of this year, will be greatly hampered throughout their entire course.

The first four years constitute a period of habit-formation, and advantage should be taken of this fact to establish proper habits of computation that will remain throughout life.

A great deal of time should still be spent in oral work—with oral problems and abstract drill work.

Written addition, subtraction, multiplication, and division (with one-figured multipliers and divisors) should still constitute a large portion of the time.

In this work which was begun in former grades, use more difficult numbers, and work for greater speed and accuracy.

The *new* work of this year consists chiefly of multiplying and dividing by two-figured numbers. Some elementary work with fractions is sometimes begun.

THE COURSE IN OUTLINE

1. Reading and writing numbers to 100,000,000.
2. Reading and writing any of the Roman numerals.
3. Complete mastery of the tables of addition and multiplication, with the corresponding subtraction and division facts.
4. Written addition and subtraction, and power to check results so as to turn in perfect papers in computation.
5. Written multiplication and division to include two-figured multipliers and divisors, and power to check work so as to turn in perfect papers in computation.
6. The application of all the processes to the solution of one- and two-step problems that come within the pupil's needs and experiences.

EXTENDING THE WORK OF FORMER GRADES

Skill once developed will be lost without constant review, and facts once known will be forgotten. It is important, then, that the child should have frequent reviews of all the fundamental facts and processes learned in former grades. Interest must be kept up by games, vital problems, contests, and rivalry. Drill without interest is practically useless. The children's general interests have widened and many of the community activities will furnish a motive for arithmetic. The operating expenses of

the home, school, and city may make an appeal. Their work in geography, such as the population and areas of countries, the heights of mountains, the length of rivers, and production of things from which we get food, clothing, and shelter all have numerical aspects and need arithmetic.

Pupils of this grade still like the playing store games, but now want the work organized upon a little larger basis. They like to play at running an express business; running a taxi line; keeping a general merchandise store; running a meat market; a musical instrument store; running a dairy farm, etc.

THE DEVELOPMENT OF MULTIPLICATION BY A TWO-FIGURED MULTIPLIER

There are three steps in the development of multiplication by a two-figured number. Each step, however, is useful in itself aside from being a step toward the finished process. The steps are: (1) multiplying by 10 or 100; (2) multiplying by multiples of 10; and (3) multiplying by any two-figured number. Each step is important and should be thoroughly developed and fixed before proceeding to the next.

MULTIPLYING BY 10 OR 100

The pupil can find such products as 3×10 , 3×100 , 4×10 , 4×100 , etc., for he knows how to multiply by any number less than 10. Since $3 \times 10 = 30$,

$10 \times 3 = 30$; and since $3 \times 100 = 300$, $100 \times 3 = 300$, etc. Thus he sees that annexing 0 to a whole number multiplies it by 10; and that annexing 00 multiplies it by 100.

Give oral drill in abstract work as 10×5 , 10×8 , 10×15 , 10×12 , etc.; also 100×3 , 100×4 , 100×15 , etc. Or, dictating such exercises, have the pupil write answers only. Do not allow such work as

$$\begin{array}{r} 3 \\ \underline{100} \\ 300 \end{array}$$

$$\begin{array}{r} 15 \\ \underline{100} \\ 1500, \text{ etc.} \end{array}$$

Give oral problems involving multiplying by 10. Thus, "At 5¢ each, how much will 10 oranges cost?" "Allowing 4 sandwiches for each, how many are needed for a party of 10?" etc.

MULTIPLYING BY MULTIPLES OF 10

The pupil can multiply any multiple of 10 by any one-figured number. Thus, he can find

$$\begin{array}{r} 30 \\ \underline{4} \\ 120 \end{array} \quad \begin{array}{r} 40 \\ \underline{6} \\ 240 \end{array} \quad \begin{array}{r} 90 \\ \underline{7} \\ 630 \end{array} \quad \begin{array}{r} 80 \\ \underline{6} \\ 480 \end{array} \quad \begin{array}{r} 70 \\ \underline{3} \\ 210 \end{array}$$

Since $4 \times 30 = 30 \times 4$, $7 \times 90 = 90 \times 7$, etc., he knows that

$$\begin{array}{r} 4 \\ \underline{30} \\ 120 \end{array} \quad \begin{array}{r} 6 \\ \underline{40} \\ 240 \end{array} \quad \begin{array}{r} 7 \\ \underline{90} \\ 630 \end{array} \quad \begin{array}{r} 6 \\ \underline{80} \\ 480 \end{array} \quad \begin{array}{r} 3 \\ \underline{70} \\ 210 \end{array}$$

From this, he sees that he may multiply by 30, by multiplying by 3 and annexing 0 to the product; by 40, by multiplying by 4 and annexing 0 to the product, etc.; for any multiple of 10 to 90.

Give much oral drill as:

20×8	30×5	40×3	50×7	50×8
20×6	30×6	40×6	60×8	90×7
20×7	30×8	40×7	50×9	60×9
20×4	30×7	40×8	70×6	50×3
20×9	30×4	40×9	80×7	70×8

As suggested throughout the book, use these facts and the process in practical problems that the child might meet. Thus, "If a class of 30 each contributes 7¢ toward refreshments for a party, how much money will they have?" "If 3 sandwiches are allowed for each of a party of 20, how many will be needed?" "If carfare is 8¢ per trip, how much will it cost to make 40 trips in coming and going to school for a month?"

DRILLS IN WRITTEN WORK

Before taking up two-figured multipliers, the pupil should have some drill in written multiplication where the multiplicand is a two-figured number and the multiplier a multiple of 10. Thus, 20×34 .

34 *First Step:* Multiply by 2, getting 68.

20 *Second Step:* Annex 0, getting 680.

680

In such work as 20×30 , the process is the same.

30 *First Step:* Multiply by 2, getting 60.
20 *Second Step:* Annex 0, getting 600.
 600

Drill Exercises

20×36	40×68	20×40	50×70
30×48	50×67	30×50	40×80
20×57	40×47	40×30	60×70
30×76	50×84	60×80	80×90
40×59	60×73	70×30	70×80
30×93	50×96	90×60	30×90

MULTIPLYING BY A TWO-FIGURED NUMBER

To multiply 34×23 involves two steps, both of which the child has had. That is, he knows how to find both 3×34 and 20×34 . And, from the meaning of multiplication, he knows that three 34's and twenty 34's make twenty-three 34's. Hence he can find the product by finding these two products and adding them. Thus,

$$\begin{array}{r}
 34 \\
 \underline{20} \\
 680
 \end{array}
 \qquad
 \begin{array}{r}
 34 \\
 \underline{3} \\
 102
 \end{array}
 \qquad
 \begin{array}{r}
 102 \\
 \underline{680} \\
 782
 \end{array}$$

Then let him see that this is shortened as follows:

34 1. Find 3×34 as if the multiplier was 3.
23 2. Then find 20×34 as if the multiplier
 102 was 20.
 680 3. Then add the two products.
782

Then, finally, let him see that instead of annexing 0 after multiplying by 2, the first product may be put in tens' column, where it would come if 0 was annexed, and thus the 0 need not be annexed. So the final form is as shown in the margin.

This development takes but little time and
 34 shows the real meaning of the process.
 23 It is well, in the drills and problems that
 102 follow, to have occasionally the answer found
 68 both by the short form and by the longest
 782 form shown above.

CHECKING RESULTS

Accuracy and speed can only be acquired through long practice. Even adults who do much computing daily do not arrive at 100% accuracy. They must use some method of checking that assures the accuracy of every computation. No computation should be considered finished until checked. In all practical uses of arithmetic, a result is of no value unless it is correct. Occasional checking is of but little value. The pupil must develop the *habit* of checking *all* computations, just as a clerk or an accountant does, before he considers the work finished.

To check addition: The only check needed in the elementary school is that of adding in reverse order.

To check subtraction: Add the difference to the subtrahend to see if it produces the minuend.

To check multiplication: Interchange multiplier and multiplicand when they have the same number of figures. Otherwise, it is best to go over the work a second time.

To check division: Multiply the quotient by the divisor and add the remainder; or, go over the work a second time.

THE TEACHING OF LONG DIVISION

Long division is the most complex and difficult of all the fundamental processes. It is the most *complex* because the child must shift from division to multiplication and subtraction, and to the bringing down the right figure. And it is the most *difficult* on account of the mental calculation necessary in estimating each quotient figure.

Moreover, it has fewer practical uses that appeal to the child in the fourth grade where custom has placed it, and hence lacks the interest that former processes had. The interest must come chiefly through the feeling of mastery in it as a process.

Many of the difficulties, however, can be avoided by a proper selection and gradation of the work. But, at best, the process is complex, and pupils will need much time and careful teaching to master it.

On account of the lack of clear practical uses of the subject in problems that enter into the life of the child before he reaches the subject of percentage

in the higher grades, the work in this grade should be confined to two-figured divisors and quotients.

The following sections will show how to minimize the difficulties of the subject.

DIVIDING BY MULTIPLES OF 10; SHORT DIVISION

The pupil knows the meaning of division and that the division facts come from multiplication. Thus, from $3 \times 4 = 12$, he knows that there are three 4's in 12, written $4 \overline{)12}$.

3

From multiplication, he can find that $3 \times 20 = 60$; hence he knows that there are three 20's in 60, written $20 \overline{)60}$. And thus from

$\begin{array}{r} 80 \\ 4 \\ \hline 320 \end{array}$	$\begin{array}{r} 70 \\ 5 \\ \hline 350 \end{array}$	$\begin{array}{r} 60 \\ 7 \\ \hline 420 \end{array}$	$\begin{array}{r} 40 \\ 6 \\ \hline 240 \end{array}$	$\begin{array}{r} 50 \\ 5 \\ \hline 250 \end{array}$	$\begin{array}{r} 60 \\ 8 \\ \hline 480 \end{array}$
--	--	--	--	--	--

he can write

$80 \overline{)320}$	$70 \overline{)350}$	$60 \overline{)420}$	$40 \overline{)240}$	$50 \overline{)250}$	$60 \overline{)480}$
4	5	7	6	5	8

And thus he sees that the 0 in each need not be considered, and he can give at sight such quotients when there is no remainder. The following drill should be used until the results can be given automatically.

WORK OF THE FOURTH YEAR

155

20) <u>60</u>	30) <u>270</u>	50) <u>350</u>	70) <u>350</u>
20) <u>80</u>	40) <u>80</u>	50) <u>400</u>	70) <u>420</u>
20) <u>100</u>	40) <u>120</u>	50) <u>450</u>	70) <u>490</u>
20) <u>120</u>	40) <u>160</u>	60) <u>120</u>	70) <u>560</u>
20) <u>140</u>	40) <u>200</u>	60) <u>180</u>	70) <u>630</u>
20) <u>160</u>	40) <u>240</u>	60) <u>240</u>	80) <u>160</u>
20) <u>180</u>	40) <u>280</u>	60) <u>300</u>	80) <u>240</u>
30) <u>60</u>	40) <u>320</u>	60) <u>360</u>	80) <u>320</u>
30) <u>90</u>	40) <u>360</u>	60) <u>420</u>	80) <u>400</u>
30) <u>120</u>	50) <u>100</u>	60) <u>480</u>	80) <u>480</u>
30) <u>150</u>	50) <u>150</u>	60) <u>540</u>	80) <u>560</u>
30) <u>180</u>	50) <u>200</u>	70) <u>140</u>	80) <u>640</u>
30) <u>210</u>	50) <u>250</u>	70) <u>210</u>	80) <u>720</u>
30) <u>240</u>	50) <u>300</u>	70) <u>280</u>	90) <u>180</u>

and so on for all of the division by 90.

These drills may be motivated by the use of games as suggested in the drills of former grades. Thus, the drills may be selected and arranged in four equal columns and used as a "baseball" game. A pupil "at the bat" being called "out on first," "out on second," etc., if he misses in any column, and scoring a "home run" if he gives all.

Pupils tire of one pupil reciting for a long period, so the bases should not contain a large number of exercises. The following has been found satisfactory.

<i>First base</i>	<i>Second base</i>	<i>Third base</i>	<i>Home plate</i>
20) <u>80</u>	60) <u>180</u>	40) <u>280</u>	30) <u>240</u>
30) <u>120</u>	40) <u>360</u>	60) <u>420</u>	40) <u>320</u>
40) <u>240</u>	50) <u>350</u>	30) <u>210</u>	50) <u>400</u>
50) <u>200</u>	20) <u>160</u>	50) <u>300</u>	60) <u>360</u>

Four such charts could be made to include *all* of the 64 such divisions. In all drills, care must be taken that *all* facts of a series are included, and not certain facts taken at random to the exclusion of others.

DIVISION BY MULTIPLES OF 10; ONE-FIGURED REMAINDERS

Following the former drill, have pupils see that

since $\begin{array}{r} 20 \overline{)80} \\ 4 \end{array}$, $\begin{array}{r} 20 \overline{)85} \\ 4, 5 \text{ remaining.} \end{array}$

That is, since there are just four 20's in 80, there must be four 20's and 5 more in 85.

In this drill, select numbers that give but one-figured remainders as:

$$\begin{array}{r} 20 \overline{)67} \end{array} \quad \begin{array}{r} 30 \overline{)158} \end{array} \quad \begin{array}{r} 70 \overline{)218} \end{array} \quad \begin{array}{r} 90 \overline{)186} \end{array}$$

$$\begin{array}{r} 30 \overline{)94} \end{array} \quad \begin{array}{r} 40 \overline{)249} \end{array} \quad \begin{array}{r} 70 \overline{)356} \end{array} \quad \begin{array}{r} 90 \overline{)275} \end{array}$$

$$\begin{array}{r} 30 \overline{)126} \end{array} \quad \begin{array}{r} 50 \overline{)352} \end{array} \quad \begin{array}{r} 70 \overline{)425} \end{array} \quad \begin{array}{r} 80 \overline{)489} \end{array}$$

$$\begin{array}{r} 40 \overline{)125} \end{array} \quad \begin{array}{r} 60 \overline{)183} \end{array} \quad \begin{array}{r} 80 \overline{)246} \end{array} \quad \begin{array}{r} 60 \overline{)425} \end{array}$$

$$\begin{array}{r} 40 \overline{)168} \end{array} \quad \begin{array}{r} 60 \overline{)245} \end{array} \quad \begin{array}{r} 80 \overline{)325} \end{array} \quad \begin{array}{r} 90 \overline{)278} \end{array}$$

These, too, may be arranged into games such as the baseball game of the last section.

DIVISION BY MULTIPLES OF 10; TWO-FIGURED REMAINDERS

Since $80 \div 20 = 4$, 95 (15 larger than 80) $\div 20 = 4$, and 15 remaining. Thus

$$\begin{array}{r} 20 \overline{)75} \\ 3-15 \text{ rem.} \end{array} \quad \begin{array}{r} 30 \overline{)76} \\ 2-16 \text{ rem.} \end{array} \quad \begin{array}{r} 40 \overline{)92} \\ 2-12 \text{ rem.} \end{array}$$

Have the pupil see that he thinks, "Three 2's in 7 and 1 remaining," then annexing the 5 of 75, he thinks, "Three 20's in 75 and 15 remaining."

Much sight drill in such tables as the following will greatly lessen the difficulty of estimating quotient

figures in long division. These drills may be used for games as in the two preceding sections.

$$20)\underline{78} \quad 30)\underline{169} \quad 70)\underline{296} \quad 90)\underline{286}$$

$$20)\underline{96} \quad 40)\underline{295} \quad 70)\underline{465} \quad 90)\underline{381}$$

$$30)\underline{115} \quad 50)\underline{176} \quad 70)\underline{397} \quad 90)\underline{476}$$

$$30)\underline{139} \quad 60)\underline{258} \quad 80)\underline{345} \quad 70)\underline{592}$$

$$40)\underline{137} \quad 60)\underline{379} \quad 80)\underline{675} \quad 80)\underline{499}$$

$$40)\underline{216} \quad 50)\underline{372} \quad 80)\underline{756} \quad 60)\underline{586}$$

FIRST LONG DIVISION

The first divisors should be those *nearly* a multiple of 10, and the quotient figures should be easy to "estimate" until the pupil gets the form of procedure, viz.: (a) estimating the quotient figure; (b) multiplying the divisor by it; (c) subtracting; and (d) bringing down the next figure of the dividend.

The dividends should be carefully made by selecting the divisor and quotient wanted and finding their product, so that there will be no difficulties that the child is not prepared for, and so that no remainders occur after the final division. Thus 21×34 would make a very good type for the first work. $21 \times 34 = 714$. Find $714 \div 21$.

If the question were "how many 20's in 71," the answer would be "3 and a remainder." So there are probably three 21's in 71. The exercises, at first, should require no further questions in order to get the right quotient. Later, further calculation will be necessary. That is, the pupil at this time disregards the 1 of 21 and the 1 of 71 and thinks, "There are three 2's in 7, hence three 21's in 71." Insist on the quotient figures being placed directly above the right-hand figure of the number divided.

First Drill Exercises

1.	2.	3.	4.	5.
$21\overline{)483}$	$31\overline{)744}$	$21\overline{)672}$	$41\overline{)984}$	$31\overline{)992}$
$31\overline{)806}$	$21\overline{)903}$	$41\overline{)656}$	$21\overline{)756}$	$21\overline{)966}$
$21\overline{)546}$	$31\overline{)558}$	$51\overline{)816}$	$31\overline{)653}$	$41\overline{)738}$
6.	7.	8.	9.	10.
$21\overline{)1323}$	$21\overline{)1554}$	$31\overline{)1116}$	$51\overline{)1071}$	$41\overline{)1722}$
$31\overline{)1054}$	$31\overline{)1302}$	$21\overline{)1764}$	$31\overline{)1953}$	$51\overline{)1785}$
$41\overline{)1312}$	$41\overline{)1066}$	$41\overline{)1435}$	$41\overline{)1395}$	$51\overline{)2142}$

SECOND STEP IN LONG DIVISION

In all exercises given above, the number obtained by dividing by the tens' digit of the divisor gives the right number in the quotient. But it is only such selected cases in which this is so. The pupil has now mastered the method of mechanical procedure. Future work is to develop ability to see the right quotient figure.

In finding $56\overline{)2688}$, the pupil knows that there are five 50's in 268. But he does not know that there are five 56's in 268. All that he knows is that there are not *more than* 5 and that there may be not more than 4. There are several courses open to him. (a) He may, on separate paper, find 5×56 and see that it is larger than 268, and thus take 4 for the quotient. (b) Or, mentally, he may be able to see that 5 is too large (by making a mental multiplication). And, (c) he may take a chance on either 4 or 5 as the quotient figure, try it, and if it is not right, take the other. The last is perhaps the most common practice among children; but, when the wrong "guess" is made, it necessitates erasing the work and trying again. The plan (a) would be a far wiser plan and would lead to neater work. But children well drilled on the preceding types— $80\overline{)246}$ or $70\overline{)295}$ —do not need to use either of these methods. With two-figured divisors, the following plan is much better. It requires more mental work but leads to much greater efficiency in all the future work of division.

- $\begin{array}{r} 48 \\ 56 \overline{)2688} \end{array}$ *First:* Think "five 50's and 18 rem."
 $\begin{array}{r} 224 \\ 448 \overline{)2688} \end{array}$ *Second:* Think "18 is less than five 6's."
 $\begin{array}{r} 224 \\ 448 \overline{)2688} \end{array}$ *Third:* Think "5 is too large. Try 4."
 $\begin{array}{r} 448 \\ 448 \overline{)2688} \end{array}$ In dividing 448. Think "eight 50's and
 $\begin{array}{r} 448 \\ 448 \overline{)2688} \end{array}$ 48 rem." Then think "48 is eight 6's;
 so 8 is right."
 $\begin{array}{r} 47 \\ 36 \overline{)1692} \end{array}$ *First:* Think "five 30's and 19 rem."
 $\begin{array}{r} 144 \\ 252 \overline{)1692} \end{array}$ *Second:* Think "19 is less than five
 $\begin{array}{r} 252 \\ 252 \overline{)1692} \end{array}$ 6's, so 5 is too large. Try 4."
 $\begin{array}{r} 252 \\ 252 \overline{)1692} \end{array}$ In dividing 252, think "eight 30's and
 $\begin{array}{r} 252 \\ 252 \overline{)1692} \end{array}$ 12 rem. 12 is less than eight 6's, so try 7."

The method given here is not the method in general use, but it is the method that I have used for many years with splendid success, and I urge teachers to try it.

Drill of More Difficult Type

$36 \overline{)2968}$	$64 \overline{)3392}$	$27 \overline{)1396}$	$94 \overline{)3572}$
$36 \overline{)2268}$	$76 \overline{)4332}$	$44 \overline{)1232}$	$56 \overline{)4256}$
$36 \overline{)2664}$	$84 \overline{)4536}$	$27 \overline{)1566}$	$76 \overline{)3268}$
$36 \overline{)3456}$	$46 \overline{)2622}$	$75 \overline{)3600}$	$64 \overline{)4672}$
$24 \overline{)1104}$	$36 \overline{)1008}$	$37 \overline{)1702}$	$84 \overline{)5376}$
$24 \overline{)1512}$	$84 \overline{)2268}$	$58 \overline{)1624}$	$36 \overline{)2052}$
$56 \overline{)2128}$	$37 \overline{)1036}$	$64 \overline{)1856}$	$76 \overline{)6612}$

These exercises have been made to give no remainder. To give this kind only may lead the pupil

to guess at the last quotient figure and write down a product like the last dividend. Hence exercises should be given that will have a remainder after the final division. These should be marked "Remainder" and not written as a fraction in the quotient, by writing the remainder over the divisor. Such fractions have no meaning to the pupil at this time. Later he will make a decimal point after the last figure of the dividend, bring down two or more 0's and continue the division, getting a decimal fraction.

DIVIDING BY THE "TEENS" AS 16, 18, ETC.

When dividing by any number from 12 to 19 inclusive, whether as but two-digit divisors, or as the left hand figures of any divisor, it is much harder to choose the right quotient figure than when the left hand figure of the divisor is 2 or more. Hence, practice with such divisors should be left until some work like that in preceding sections has been given.

Thus, to divide 630 by 18.

Here dividing 6 by 1 gives no clue to the right quotient figure. To think "about 20" for 18, in this case gives the right first quotient figure. But such an estimate would give but 4 instead of 5 in the division of 90. The only thing to do here is to "use judgment" in selecting possible quotients

$$\begin{array}{r} 35 \\ 18 \overline{)630} \\ \underline{54} \\ 90 \\ \underline{90} \\ 0 \end{array}$$

and multiply out (mentally or on separate paper) until the right one is found.

A second example: Divide 736 by 16.

Here $7 \div 1$ gives no clue. And $7 \div 2$ gives a number too small. Also the same trouble is seen in $9 \div 1$ and $9 \div 2$.

It is thus seen that the pupil will have to "try out" several possible quotients until he finds the right one. To do this well he must have much practice in multiplying at sight all numbers from 12 to 19, by all one-figured numbers. A table of these should be used a few minutes daily.

Drill Table (give products at sight)

2×12	2×13	2×14	2×15	2×16	2×17	2×18	2×19
3×12	3×13	3×14	3×15	3×16	3×17	3×18	3×19
4×12	4×13	4×14	4×15	4×16	4×17	4×18	4×19
5×12	5×13	5×14	5×15	5×16	5×17	5×18	5×19
6×12	6×13	6×14	6×15	6×16	6×17	6×18	6×19
7×12	7×13	7×14	7×15	7×16	7×17	7×18	7×19
8×12	8×13	8×14	8×15	8×16	8×17	8×18	8×19
9×12	9×13	9×14	9×15	9×16	9×17	9×18	9×19

DIVIDING BY NUMBERS NEARLY A MULTIPLE OF 10

In dividing by such numbers as 58 or 59, some consider it easier to get the quotient by thinking of them as "nearly 60."

$$\begin{array}{r} 36 \\ 58 \overline{)2088} \\ \underline{174} \\ 348 \\ \underline{348} \end{array}$$
 Thus, think "58 is nearly 60. There are three 6's in 20." Write 3. While this gives the right quotient figure in $208 \div 58$, such an estimate does not give the right figure in $348 \div 58$.

The plan shown in a preceding section, of thinking "four 50's and 8 rem.; 8 is smaller than four 8's; so use 3," seems better than the method used by so many.

Even in an example like $2655 \div 59$, it is better to use the general method of estimating than to think "about 60."

While "about 60" would give the first figure more easily and quickly than to

$$\begin{array}{r} 45 \\ 59 \overline{)2655} \\ \underline{236} \\ 295 \\ \underline{295} \end{array}$$
 think "five 50's and 15 rem.; so 5 is too small," it would not give the second figure. To think "five 50's and 15 rem., which is too small, write 4," and in $295 \div 59$ to think "five 50's and 45 rem. $5 \times 9 = 45$, 5 is right," requires more mental calculation than to write down, multiply, and thus "try out" any quotient that suggests itself, but it is more economical and gives valuable drill in mental calculation.

NOT ALL DIFFICULTIES CAN BE OVERCOME

It must have been seen from the preceding discussions that either written or mental calculations must be done in many cases before the right quotient figures can be chosen. It is not to be expected that

pupils will always select the right quotient figure. Expert computers do not. But pupils should be carefully trained in the method shown here in order that wrong selections will be reduced to a minimum.

When using divisors of more than two figures, the method is the same, using but the two left hand figures (noting, however, those that follow). Thus to divide 89,436 by 348.

$ \begin{array}{r} 257 \\ 348 \overline{)89436} \\ \underline{696} \\ 1983 \\ \underline{1740} \\ 2436 \\ \underline{2436} \end{array} $	Using but 34, think "two 30's and 29 rem. So 2 is right." Then think "six 30's and 18 rem., which is less than 6×4 . Use 5." Next think "eight 30's and 3 rem. Too small, use 7." Except in very close cases, there is no need of writing down the wrong figure.
--	--

DIVISION WITH ZEROS IN THE QUOTIENT

The pupil will need guidance in what to do when a zero occurs in the quotient. Hence such exercises must not be thrown in with the other work until the child has consciously encountered them and been shown how to handle them. But little explanation is needed.

$ \begin{array}{r} 205 \\ 43 \overline{)8815} \\ \underline{86} \\ 215 \\ \underline{215} \end{array} $	Simply say that to show that 21 will not contain 43 a zero is written in the quotient above the 1 and the next figure brought down. To convince the pupil that this gives the right answer, let him multiply 43 by 205.
---	---

Drills to Give Skill in Long Division

Since the main difficulty in long division is to estimate the quotient figure, sight drills in making such estimates are valuable. The pupil can thus find fifty or one hundred quotient figures in the same time that he could actually divide three or four numbers giving two quotient figures each. Such drills should not be even divisions, but such exercises as the following:

$$21 \overline{)82} \quad 31 \overline{)165} \quad 46 \overline{)348} \quad 53 \overline{)319}$$

$$34 \overline{)146} \quad 34 \overline{)196} \quad 54 \overline{)468} \quad 67 \overline{)198}$$

$$25 \overline{)138} \quad 42 \overline{)256} \quad 36 \overline{)192} \quad 73 \overline{)293}$$

$$42 \overline{)165} \quad 53 \overline{)368} \quad 65 \overline{)384} \quad 84 \overline{)358}$$

$$54 \overline{)261} \quad 45 \overline{)297} \quad 84 \overline{)426} \quad 76 \overline{)304}$$

$$63 \overline{)375} \quad 64 \overline{)385} \quad 93 \overline{)398} \quad 83 \overline{)416}$$

$$84 \overline{)336} \quad 76 \overline{)375} \quad 76 \overline{)457} \quad 78 \overline{)392}$$

$$76 \overline{)297} \quad 83 \overline{)426} \quad 85 \overline{)336} \quad 56 \overline{)435}$$

$$72 \overline{)298} \quad 93 \overline{)375} \quad 94 \overline{)296} \quad 64 \overline{)449}$$

The Uses of Long Division

The child's problems in which long division is needed are rare. However, a few types may be found, illustrated by the problems that follow.

1. In making a paper chain to trim a Christmas tree, the pupils decided to make 180 links. If 15 children volunteer to make them, that will be how many for each?

2. If Charles is saving 65¢ per week to buy a coaster, how long will it take him to save enough to buy a coaster costing \$7.80?

3. By saving 85¢ per week, how long will it take John to save enough to buy a bicycle costing \$17.85?

4. A fourth grade class wants to raise \$65.25 to buy a phonograph. They are going to give an entertainment. How many tickets at 45¢ each will they have to sell to raise all the money?

5. A class picnic for 34 cost \$15.30. How much was that for each?

6. Mary is reading a book of 252 pages. If she reads 18 pages per hour, how long will it take to read the book?

7. John's father drove his car 384 miles one month and used 24 gallons of gasoline. He asked John to find how many miles he got per gallon. Can you find it?

A FIRST TREATMENT OF FRACTIONS

Instead of leaving the first and complete treatment of fractions until the fifth grade, as we did under a rigid adherence to the topic plan of teaching, we now develop certain elementary notions of a fraction during the first four grades. Thus the child

learned the meaning of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., of multiples of 2, 3, 4, etc., when first studying division. And as need of it arose, further meaning and use was made of a fraction. He thought of it as one or more of the several equal parts into which some object had been divided.

THE NOTION OF A FRACTION

When any object or group of objects is divided into a number of equal parts, the child knows that each part is "one of two equal parts of the object or group"; "one of the three"; "one of the four"; etc., according to the number of divisions. Thus dividing some object, or some line drawn upon the blackboard, into five equal parts, the pupil can describe one or more of them as "one of the five equal parts of —"; "two of the five equal parts of —"; etc. He should then see the expression $\frac{1}{5}$, $\frac{2}{5}$, etc., as mere symbols for these longer expressions, and the symbols should be interpreted by him as representing those words. Thus he should read the symbol $\frac{5}{8}$ as "5 of the 8 equal parts of something," as $\frac{5}{8}$ yd. is "5 of the 8 equal parts of a yard." He learns the shorter way to read $\frac{5}{8}$ as "five-eighths." There is little value this year in having him learn the names "numerator" or "denominator." The valuable thing is to have him see the real meaning of the symbol.

RELATIONS EXPRESSED AS FRACTIONS

One of the first and most important uses of a fraction is to express the relation of one group or quantity to a larger one. Thus if John has 5 marbles and loses 1 of them, he loses "one of the five" or $\frac{1}{5}$ of them; if he loses "two of the five," he loses $\frac{2}{5}$ of them, etc.

A pint is "one of the two equal parts of a quart" or $\frac{1}{2}$ qt. A peck is "one of the four equal parts of a bushel" or $\frac{1}{4}$ bu. An inch is "one of the twelve equal parts of a foot," or $\frac{1}{12}$ ft. An ounce is "one of the sixteen equal parts of a pound" or $\frac{1}{16}$ lb.; five ounces are "five of the sixteen equal parts of a pound," or $\frac{5}{16}$ lb.; and so on.

PROCESSES WITH FRACTIONS

There will be but few, if any, activities of the child in this grade that require adding, subtracting, multiplying, or dividing fractions. Those fractions that are needed will be so simple that they can be readily handled by the pupil from his knowledge of similar processes with whole numbers if he understands the meaning and notation of a fraction as shown above. He may need, however, to find a fractional part of a whole number, such as $\frac{3}{4}$ of 24 or $2\frac{3}{4} \times 48$, etc. But this gives no trouble, for he has learned in the third grade how to find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., of numbers within the tables.

Knowing, then, how to find "one of the four equal parts of a number," he will know how to find "three of the four equal parts of it," by dividing and multiplying.

FINDING TWO OR MORE OF THE EQUAL PARTS OF A NUMBER

From the meaning of $\frac{3}{4}$, to find $\frac{3}{4}$ of 24, the child would, of course, first divide by 4 and then multiply by 3. Thus,

$$\begin{array}{r} 4 \overline{)24} \\ \underline{6} \\ 3 \\ \underline{18} \end{array}$$

But in future work when there will be remainders after the division, it is more economical to multiply first. So, through examples, the pupil should see that to multiply first and then divide gives the same result, and that unless he can see the quotient easily, it is best to multiply first. In multiplying by a mixed number, it is much better to follow the second plan.

96 Thus to find $2\frac{3}{4} \times 96$.

$$\begin{array}{r} 2\frac{3}{4} \\ \hline 4 \overline{)288} \end{array}$$

Here he first finds $\frac{3}{4}$ of 96 by multiplying 96 by 3 and dividing the product by 4.

72 This gives the first partial product.

$$\begin{array}{r} 192 \\ \hline 264 \end{array}$$

He then finds 2×96 and adds the two partial products.

STANDARDS OF SKILL

There are well-known standard tests in computation that may be used during the fourth year, by which skill in computation may be measured. The Curtis tests and the Woody tests are best known. The Curtis tests are tests in the four fundamental processes with whole numbers. The Woody tests are graded from easy to difficult exercises in the four fundamental processes with whole numbers, fractions, decimals, and denominate numbers.

These tests, however, are not available to all, and at all times. So there should be a more simple type of test that is available for all, and at all times. The following, then, are submitted as a reasonable attainment for the end of the fourth year.

In addition: (1) Twenty single columns of six figures each in 5 minutes; or (2) Ten exercises of four three-figured numbers in 5 minutes.

NOTE.—The exercises should be carefully made so as to bring in all the primary combinations instead of the same combinations many times. The number to be done in 5 minutes is the speed attainment. Teachers should expect about 90% or 95% accuracy. Interpret each of the following standards in much the same light as this note suggests for addition.

In subtraction: Fifteen four-figured numbers, (no exercises to contain more than two “borrowings”), in 5 minutes.

In multiplication: (1) Ten two-figured numbers multiplied by a two-figured number; or, (2) Six

three-figured numbers by a two-figured number in 5 minutes.

In division: Six four-figured numbers divided by two-figured numbers, giving a two-figured quotient and a remainder, in 5 minutes.

These tests, however, measure only the mechanical aspects of the subject—the tools of arithmetic. In striving to attain these standards in computation, the other aspect of the subject should not be slighted. Just as abilities to spell and to write are of no value unless we have thoughts to express through these abilities, so abilities to compute are of no value unless we can interpret a problem and know what processes to apply to its solution.

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